REINFORCED CONCRETE ARCH BRIDGE
SHILLER PARK, ILLINOIS

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ARMOUR INSTITUTE OF TECHNOLOGY

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DESIGN OF A TWO SPAN
DOUBLE TRACK REINFORCED CONCRETE
TROLLEY CAR ARCH BRIDGE
AT SHILLER PARK, ILLINOIS

A THESIS

Presented by

Roswell P. Pearce.

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to the

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of the

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INTRODUCTION.

In view of the fact that is highly desirable to write a thesis upon a subject which may be of practical value, this reinforced concrete arch has been dhosen.

The bridge as designed is a proposed structure to replace an old Bowstring Girder across the Desplaines River at Shiller Park, Illinois, in the township of Leyden.

At the present time steps have already been taken by the commissioners of the township toward putting up a new bridge.

The road at this point is a continuation of Irving Park Boulevard, Chicago, Ill., and since it is probable that electric cars will run over it in the near future, the structure has been designed as a double track trolley car bridge.

The soil is clay and, in the design, the bearing on the clay is taken at 2-1/2 tons while on the piles, which are necessary, as 18 tons per pile.

There is about three feet of sand in the river bed over the clay.

Figure 1 shows the topographic map of the immediate vicinity of the bridge. It is intended to place the

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West end in the position occupied by the west end of the old bridge but to move the East end about 6 feet South so as to take out the bend in the road.

The water way has been taken amply large to provide for the spring floods inasmuch as careful measurements have been made on the Lake Street bridge, a similar 3 span structure farther down the river.

Plates 2 and 3 show the design of the arch proper.

Plate 4 shows the forces acting on the abutments.

Plate 5 gives the steel used in the structure with sections of the wing walls, spandril walls, piles, etc.

Plate 6 shows the finished elevations of the bridge.

Reuterdahl's method for reinforced concrete arches is the method used in the design.

DESIGN OF THE REINFORCED CONCRETE ARCH.

LOADING.

Dead Load.- Actual weight of material in the structure. Completed on the basis of 150 lbs. per cu. ft. for concrete and 120 lbs. per cu. ft. for backing above arch. The ratio $\frac{120}{150} = \frac{4}{5}$; consequently, for convenience, we can draw a line of reduced backing, (Plate 3), at a distance above the crown equal to 4/5 of crown distance between surface of roadway and extrados curve. Then we can multiply every cubic ft. of material below the line of reduced backing by the single constant 150 to find the total dead load. This gives somewhat more than the actual load, for the actual line of reduced backing is not a straight line but a curve approaching the extrados curve at the springing more than the straight line.

Live load:- The live load will be computed on the basis of 200 lbs. per square ft. of the roadway and sidewalks.

Proportioning of the Arch,

(a) Crown Thickness

(F. F. Weld's Formula.)

Let d = depth of arch at crown in inches.

L = the clear span in feet.

w = live load in pounds per square foot uniformly distributed.

p = weight of dead load above crown of the arch per square foot in pounds.

then, $d = \sqrt{L} + 0.1\,L + 0.005\,w + 0.0025\,p$

(D. B. Luten's Formula.)

Let t = thickness of crown in inches.

S = span in feet.

r = rise from springing to crown in feet.

F = fill over crown of extrados in feet.

L_1 = live load (uniform) in pounds per square foot.

L_m = moving load that will be concentrated on single track or single roadway, over entire span in tons of 2,000 lbs.

then $t = \dfrac{3\ S^2\ (r + 3F)}{4{,}000\ r - S^2} + \dfrac{L_1 S^2}{30{,}000\ r} + \dfrac{L_m\quad 5\,r\,)}{r\quad 150\ r} + 4.$

(b) Construction of the Arch. (Plate 2).

The depth at the crown in inches calculated from Weld's Formula = 17.35, from Luten's 13.26. A crown thickness, bc, of 16 inches will be used. Lay this off on Fc continued where Fc is the rise of the intrados curve.

Construct the portion of the ellipse Sec passing through points S and c.

On SS_1, the line joining the springings, take $SV = \frac{1}{10}\,L$ where L = length of span = 72 feet.

Erect a perpendicular at V to SS_1 until it intersects the ellipse in e.

Draw line Sc. Bisect angle ScF by line Ke. Through point e erect a perpendicular BK to line Kc. Through points S and c pass an arc of a circle, center at R_s on line cF extended. (Line cF is perpendicular to SS_1). Along the line BK there is intercepted the distance De between the ellipse and the arc just drawn. Bisect De in point C^1. Take on line BK, the length C^1A = AB = cb the crown thickness. Bisect cb at a. Draw AP perpendicular to Fc.

Let P_a = m and

2 AP = 1 then

$$R_n = \frac{\overset{R_n}{\cancel{m}} + (1^2 \div 4)}{2\ rn}$$

where R_n = radius of neutral axis

and rn = rise of neutral axis.

It is seen from the above that

$$rn = 2R_n \pm \frac{(4R_n^2 - 1^2)\ 1/2}{2}$$

With length R_n as a radius and point R_n as a center describe an arc of a circle passing thru point A and a. Line Aa is then the neutral axis.

Through points B and b pass an arc of a circle whose radius R_e may be found from the above equation by substituting the corresponding rise and span. This determines the extrados curve.

Divide the distance A a along the neutral axis into four equal parts finding points v, u and r. Through these points draw radial lines to point R_n. From the points v, u and r take v v_2 = v v_1; u u_2 = u u_1; and r r_2 = r r_1. Measure all these distances radially.

We found one arc R_1 passing through points C, r_2, v_2 and C'.

The curve is completed with an arc of short radius. This completes the intrados curve.

Proportioning of the Backing.

In highway bridges carrying electric cars+in country bridges the distance between the maximum surface elevation of roadway and the extrados curve at the crown may be determined as follows:

If

t = thickness of crown

F = distance between maximum elevation of roadway and extrados curve at crown, then

F = 0.9 t

This gives the amount of backing required.

Conditions of Calculation.

15. Conditions of Calculations.- Modulus of elasticity of concrete, 1,500,000 lbs., and modulus of elasticity of steel, 30,000,000 lbs.

Maximum compression allowed on concrete in the arch exclusive of temperature stresses, 500 lbs. per sq. in.; including stresses due to 40° F. variation in temperature, 600 lbs. per sq. in. Slabs, girders, beams, floors, walls and posts in subways and girder bridges shall have a safety factor of 4 in one month.

Maximum tension allowed on concrete in arches, exclusive of temperature stresses, 50 lbs. per sq. in. (Best practice is to allow no tension for concrete.) In arches, including stresses due to 40° F. variation in temperature, 75 lbs. per sq. in. In slabs, girders, beams, floors, walls and posts, 0 lbs. per sq. in.

Maximum shear allowed on concrete, 75 lbs. per sq. in.

-Maximum stress allowed on steel in arches.

Safe working stress, safety factor 4 = 15,000 lbs. sq. in.

Safe working stress, safety factor 5 = 12,000 lbs. per sq.in.

Safe working stress, safety factor 6 = 10,000 lbs. per sq.in.

In a true combination design of steel and concrete $f_s + f_c = 20$ for values of E_s and E_c assumed, hence the stress on the steel should not exceed 20 f_c.

Hence if the maximum stress for concrete (exclusive of temperature) is 500 lbs. per sq. in., then the maximum allowable stress on the steel is 10,000 lbs. per sq. in., giving a safety factor of 6.

In slabs, girders, beams, floors, walls, subjected to transverse stress, the steel should be assumed to take the entire tensile stress without aid from the concrete, and shall have an area sufficient to equal the compressive strength of concrete composed of 1 part Portland cement,3 parts sand and 6 parts of broken stone, of the age of six months.

In walls and posts subjected to compression only, no allowance shall be made for the strength of the imbedded steel, which is to be used merely as a precaution against cracks due to shrinkage or changes in temperature.

Design of the Arch Ring.-

Data:- Full-barrel arch ring; clear span of intrados curve = 72 ft., rise of intrados curve = 13 ft. 6 inches; thickness at crown = 16 inches = 1.33 feet; live load = 200 lbs. per sq. ft. on roadway and sidewalk; dead load = 150 lbs. per cu. ft. for concrete and 120 lbs. per cu. ft.for backing.

Order of Procedure.- (a) Construct trial arch. Mark lengths of all radii on drawing.

(b) Proportion arch so that $\frac{S}{D^3}$ = constant.

(c) Location of points a and loads P.

(d) Construction of equilibrium pologon. (1) Graphical method.

(e) Determination of position of true closing line of equilibrium pologon.

(f) Determination of position of true closing line of neutral axis regarded as an equilibrium pologon.

Conditional equations pertaining to true closing line of neutral axis.

(a) $\Sigma(M) = 0$ hence $\Sigma(k\ a) = 0$

(b) $\Sigma(M\ x) = 0$ hence $\Sigma(k\ a\ .x) = 0$

(g) Development of pressure curve of arch by superposition of equilibrium polygon on arch.

(1) Conditional equation b $\Sigma(M_y) = 0$ must be satisfied,

(2) Determination of true pole-distance H.

(3) Construction of pressure curve.

We will now consider, in detail, the individual steps in the "Order of Procedure."

(a) Construct Trial Arch. (1) Determine first the thickness at the crown by both Weld's and Luten's formulas.

Adopt an integral number of inches for crown thickness (2) Construct trial arch by method explained and shown by Plate 2. Place such data as lengths of all radii, rise of intrados and neutral axis, value of subtended angles, etc., on the drawing. Endeavor to get as large a rise for the neutral axis as the conditions will possibly allow.

(b) Proportion arch so that $\frac{S}{D^3}$ = constant. Since an arch has been adopted this merely reduces to a subdivision of the neutral axis into lengths S_1, S_2, S_3 etc., measured along the neutral axis so that $\frac{S_1}{D_1^3} = \frac{S_2}{D_2^3}$ $\frac{S_3}{D_3^3}$ etc. = constant.

To facilitate this work, which is somewhat laborious, the depth of the arch ring (in feet) for every foot of length of one-half of the arch from the springing line to the crown may be tabulated as in Table II.

Suppose S_{20} = 12 feet (Trial assumption) (Table I). Find depth D_{20} of arch at mid-point of S_{20} i.e., at a point 6 feet from the springing. In table II. find depth at 6 feet from springing = 2.5 feet.

$\frac{S_{20}}{D^3_{20}} = \frac{12}{(2.5)^3} = .768$ - proceed in like manner.

The work in detail is shown in Tables I and II. Five trials were necessary before proper subdivision of the neutral axis was effected.

Not less than twenty subdivisions of the neutral axis should be used.

The length of the neutral axis is computed from the radian formula and found to be 75.6 feet since the angle of R_n = 60° 21'.

(c) Location of Points a and Loads P.- (1) Location of Points a. Beginning at one of the springing lines, (say at the left) bisect in succession the lengths S_{20}, S_{19}, S_{18}, S_{17}, . . . S_3, S_2 and S_1 along the neutral axis. Label the respective mid-points of the section a_{20}, a_{19}, a_{18}, a_{17}, ... a_3, a_2, and a_1.

Through points a_{20} . . . . a_1 draw vertical lines. Since the arch is assumed symmetrical, the center line A B bisects the length a_{11} a_{10}. Using these two parts as independent sections we have between the vertical lines from a_{20} to a_1, twenty sections leaving a portion of the arch from a_{20} to the left springing and a similar portion from a_1 to the right springing. It should be remarked that the points a_{20} . . . . . a_1 are the points at which the depths D, given in Table II., were measured for the corresponding section S.

If then we use the vertical lines through the points a as boundary lines for our load sections, the radial depth d at the boundary of each section will satisfy the condition that $\frac{S}{D^3}$ = a constant.

A load section will then consist of the volume inclosed between two longitudinal vertical planes 1 ft. apart and the two tranverse vertical planes which pass through any two adjoining points a; and between the plane of the reduced backing and the surface of the intrados, i.e., the soffit. This is true in every case except at the crown where the transverse vertical plane through the center produces two sections instead of one, i.e., the section from the center line to a_{11} and the section from the center line to a_{10}. By this means, as will immediately be apparent, we can locate the load on each side and near the center line and yet avoid the undesirable location of a load at the center line.

(2) Location of Loads P.- We will now concentrate the weight included in each load section at the center of gravity of the corresponding section thus obtaining the location of the load or force lines P.

As an illustration of the method employed, take

the load section M.B D E. Regard this section as a trapezoid, i.e., regard line M B as a straight line.

If in the trapezoid M B D E,

a = E M (the longe side)

b = D B (the short side)

c = altitude of the trapezoid (E D in M B D E)

Then

$$\text{Area trapezoid} = 1/2\ (a + b)\ c.$$

Since the tranverse thickness of all the load sections is 1 ft.,

$$\text{Volume trapezoid} = 1/2\ (a + b)\ c.$$

If

W = dead load per load section.

W_1 = live load per load section.

then

$$W = 1/2\ (a + b)\ c\ (150) \text{ hence}$$

$$W = 75\ (a + b)\ c.$$

The live load distributed on any load section as at E D, must equal the area of a rectangle having E D for one side and 1 ft. for the other (for the longitudinal vertical planes are 1 ft. apart) multiplied by the live load (uniform) assumed per square foot, hence if w_1 is the live load in pounds per square foot,

$$W_1 = w_1 .\ c.$$

For the present case where w_1 = 200 lbs. per sq. ft. W_1 = 200 c.

In Table III the entire work is shown in detail. In column 4 if 75 is used instead of 150 then (a ± b) must be used instead of 1/2 (a b) in column 1.

To produce maximum moments we concentrate the live load on one-half of the arch - the left half in this case.

In the last column Table III we give the total summation for live load and dead load.

As we approach the crown the trapezoidal load sections approach the rectangular form and, therefore, it will be close enough to locate the loads P for such sections at the mid-points of c, the altitudes of the sections.

The location of the load or force lines P as well as their magnitudes having been determined their effects at points, a, the mid-points of the sections S must be determined.

The development of the entire theory depends upon this fact that the effects are to be assertained for these mid-points a hence the error must not be made of making calculations for points along the load and force lines.

To the left of point a_{20} and to the right of a_1 we

have load sections whose force lines P E, at the left and right respectively, should be determined as above.

(d) Construction of Equilibrium pologon. Graphical method. ~~(2)~~

(a) Assume trial horizontal thrust H_1, to act between two loads near the crown, say P_{11} and P_{12}. This necessitates that the ray H_1 in the ray diagram must be a horizontal line O'D perpendicular to the load line A C at point D where load P_{11} ends and load P_{12} begins. Point D gives us a starting point on the load line A C from which to lay off the loads P in succession to some arbitrarily selected scale of a certain number of pounds to the inch; in this instance, 5000 lbs. = 1 in.

The loads for the portion of the arch to the right must be scaled on A C toward A (above O'D) and those to the left, below O'D.

(b) Assume a trial pole-distance (trial horizontal thrust H_1) equal to about 1/2 sum total of all the loads, i.e., 1/2 of 71,568 or in round numbers 35,000 lbs. By using about one-half the value of the total load for trial H_1 the angle at O' becomes nearly a right angle which is desireble. Lay off on line D O' a length equal to 35,000 lbs. to the scale of 5,000 lbs. per inch, hence make D O' = 7 ins. thus determining the position of the trial pole O'.

In the equilibrium polygon the line of the polygon which is drawn between the loads P_{11} and P_{12} must be parallel to the ray of the ray diagram which is drawn to point D where load P_{11} ends and P_{12} begins; therefore the polygon line between P_{11} and P_{12} must be paraiell to ray O' D or to the trial horizontal line H_1. Hence it follows that this side of the polygon, i.e., between forces P_{11} and P_{12} must be a horizontal line. Draw therefore between P_{11} and P_{12} the line h h which we extended to both sides. Where h h intersects load line P_{12} begin another side of the polygon. Make it parallel to the ray drawn from O' to the point where P_{12} ends and P_{13} begins on the load line A C of the ray diagram. Where this last found side of the polygon intersects P_{13} begin another side and make it parallel to the ray which passes to the point of junction of P_{13} and P_{14} of the ray diagram. Proceed in this way for all the sides of the polygon until P E at the left is reached where polygon side B'C must be parallel to the outside ray 'O C. Similarly for the right hand side of the polygon.

The equilibrium polygon may be drawn in the reverse order. Beginning at point C of the load line A C, let the outside ray C 9' itself be coincident with the side

of the polygon thus obtaining C B'. From B' draw B' F parallel to ray B O' etc., continuing until the load line P_{12} is reached when the side of the polygon between P_{12} and P_{11} is drawn horizontally as h h.

(e) Determination of Position of True Closing Line of Equilibrium Polygon.-

(1) Conditional Equations Pertaining to True Closing Line.

(a) $\Sigma(M) = 0$ hence $\Sigma(v\ m) = \Sigma(b\ v)$

(b) $\Sigma(M\ x) \neq 0$. Resultant of $\Sigma(b\ v)$ must equal and coincide with resultant of $\Sigma(vm)$.

(2) Determination of Magnitude of Resultant R.

(3) Determination of Location of Resultant R.

(4) Location of True Closing Line to Equilibrium Polygon so that the sum of its ordinates to $v_{20}\ v_1 = R$, acting r feet to left of Center Line. (a) Location and Magnitude of Trial T_0.

(b) Location and Magnitude of Trial T_1.

(c) Determination of Position of True Closing Line.

(1) With the equilibrium polygon drawn by the above methods extend vertical lines from a_{20}, a_{19}, ... a_1 on the neutral axis of the arch until they intersect

the polygon in points b_{20}, b_{19}, ... b_1 and the line $v_{20}v_1$ (which should be drawn) in points v_{19}, v_{18} ....v_2.

(2) Determination of Magnitude of Resultant R. Let R be the resultant of the b v ordinates regarded as forces, then R must equal their sum, hence

$$R = \Sigma(bv) = \Sigma(vm)$$

The sum of the b v ordinates may be found by scaling from the diagram, usingthe scale of distance adopted for the drawing.

In Table IV, columns 3 and 4, the values of the ordinates vb are given and their sum $\Sigma(bv)$ H R = 129.19.

The resultant R=129.19 must act to the left of the center line for the live load is distributed over the left half, hence making the loading for the left side greater than for the right side. The reverse will be true if the live load acts on the right half of the arch.

(3) Determination of Location of Resultant R. - Let us now take moments about the center line of the ordinates b v regarded as forces. The lever arm for $v_{20}b_{20}$ will equal the lever arm for v_1b_1; let it be designated by z_1. The moments will have opposite signs being on opposite sides of the center line. We obtain for these two forces:

$$(v_{20}b_{20})\ z_1 - (v_1\ b_1)\ z_1 = 0$$

since $v_{20}b_{20} = 0 = v_1b_1$.

Similarly for the symmetrically located pair, $v_{19}b_{19}$, v_2b_2 having lever arms each equal to z_2 we obtain,

$$(v_{19}b_{19})\ z_2 - (v_2b_2)\ z_2 = 13.28$$

In the same manner the other terms may be obtained, as shown in detail in Table IV.

Taking the sum of all these terms we obtain the sum total of all the moments of all the vb ordinates about the center line. Let this sum total of moments vb be designated as $\Sigma(vb.z)$ then

Fig. 36.

$$\Sigma(vb.z) = R.\ r$$

and $$r = \frac{\Sigma(vb.z)}{R}$$

where r is the distance from the center line that the resultant R acts.

But since $\Sigma(vb.z) = 56.16$ (by Table IV), hence

$$r = \frac{\Sigma(vb.z\ )}{R} = \frac{56.16}{129.19} = \underline{.4345}$$

Therefore the resultant R acts .4345 ft. to the left of the center line.

(4) Location of True Closing Line to Equilibrium Polygon so that Sum of Its Ordinates to $v_{20}v_1 = R$, acting r feet to Left of Center Line.

(a) Location and Magnitude of Trial T.

(b) Location and Magnitude of Trial T_1.

(c) Determination of Position of True Closing Line.

(a) Having drawn the trial closing line n n_1 for the equilibrium polygon so that $v_{20}n = v_1n_1 = \frac{R}{N} = \frac{129.19}{20}$ $= 6.45$ draw the straight line n v_1 dividing the quadrilateral v_{20} n n_1 v_1 into the two triangles v_{20} n v_1 and n v_1n_1.

Let

Resultant of ordinates of triangle v_{20} n v_1 = Trial T.

Resultant of ordinates of triangle n v_1 n_1 = Trial T_1.

Resultant of ordinates of quadrilateral v_{20} n n_1 v_1 = R_1.

Sum of **ordinates** quadrilateral v_{20} n n_1 v_1 = sum of ordinates of triangle v_{20} n v_1 + sum of ordinates of triangle n v_1 n_1

Hence

$$R_1 = \text{Trial } T + \text{Trial } T_1.$$

The ordinates of the triangle v_{20} n v_1 are of the type v d. The resultant of these ordinates treated as forces = Trial T

Hence

$$\Sigma(v\ d) = \text{Trial T}$$

By Table V

$$\Sigma(v\ d) = 64.47 = \text{Trial T.}$$

We can locate the position of Trial T by the same process as was used for the location of R.

Hence taking moments about the center line of the ordinates v d of triangle v_{20} n v_1 and using the same lever arms z as for R we obtain,

$$\Sigma(v\ d.z) = 474.12$$

As the sum of the v d ordinates of triangle v_{20} n v_1 to the left of the center line is greater than the sum to the right, Trial T must act to the left of the center line.

Let t = distance that Trial T acts to left of center line, then $\Sigma(v\ d.z) = \text{Trial T} \cdot t.$

hence

$$t = \frac{\Sigma(v\ d\ .\ z)}{\text{Trial T}}$$

or

$$t = \frac{474.12}{64.47} = 7.35 \quad \text{See Table V.}$$

(b) The location and magnitude of Trial T_1 may be obtained in a similar manner.

The ordinates of triangle $n\ v_1\ n_1$ are of the type $n\ d$. As above,

$$\Sigma(n\ d\ .\ z) = \text{Trial } T_1\ t'$$

and

$$t' = \frac{\Sigma(n\ d\ .\ z)}{\text{Trial } T_1}$$

where t' is the distance that T_1 acts from the center line.

(c) Determination of Position of True Closing Line.-

$$\text{True } T = \frac{Rl^1}{2t} = \frac{129.19\ (7.7845)}{14.7} = 68.41$$

$$\text{Trial } T = 64.47$$

$$V_2 n = 6.46$$

$$V_{20}\ M = \frac{68.41}{64.47}\ (6.46) = 6.856$$

$$\text{True } T_1 = \frac{Rl}{2t} = \frac{129.19\ (6.9155)}{14.7} = 60.78$$

$$\text{Trial } T_1 = 64.63$$

$$V_1\ n_1 = 6.46$$

$$V_1\ M_1 = \frac{60.78}{64.63} \times 6.46 = 6.075$$

We see by Table VI, column (3) that the condition that $\Sigma(M) = 0$ is satisfied); Similarly for the remaining conditions.

(f) Determination of Position of True Closing Line if Neutral Axis Regarded as an Equilibrium Polygon.

(f) Determination of Position of True Closing Line of Neutral Axis Regarded as an Equilibrium Polygon.- (1) If we regard the neutral axis a_{20} a_{10} a_1 of the arch as an equilibrium polygon we must find a closing line kk so located that the conditions which held good for the neutral axis. Hence the following conditions must be satisfied:

$$(a) \quad \sum(M) = 0$$

$$(b) \quad \sum(M\ X) = 0$$

Let the ordinates of the neutral axis, Y_{20} ....Y_{10} .....Y_1 from a_{20} .....a_{10}.......a_1 to horixontal line OO_1 be designated as of the type y.

Let us, in a way analogous to our procedure with the equilibrium polygon, regarde the ordinates y as forces.

The resultant of the ordinates y regarded as forces must coincide with the center line, for these ordinates are symmetrically disposed, and respectively equal, at equal distances from the center line.

The coordinates of the points a about point 0 as an orgin will be, as above, designated as x and y. Suppose that the closing line kk is not parallel to the horizontal line OO_1.

Then the force ordinates y on one side of the crown center line will be greater than on the other side and the resultant will be located on the same side of the center line with the greater force ordinates. Since for $\Sigma(M\ x) = 0$ the resultant must pass through the center line, for symmetrically located force ordinates y are equal, hence kk must be parallel to OO_1 and therefore also a horizontal line and figure O k k O_1 becomes a rectangle. This becomes clearly evident if we take moments about the center line itself.

For $\Sigma(M) = 0$ the ordinates of rectangle O k k O_1 must equal the ordinates of neutral axis segment O a O_1.

Hence if the ordinates from line OO_1 to k k be designated as kO and from OO_1 to the neutral axis as y we must have the following:

$$\Sigma(kO) = \Sigma(y), \text{ which gives}$$

$$\Sigma(y) - \Sigma(kO) = 0 \text{ and}$$

$$\Sigma(y - k\ O) = \Sigma(k\ a) = 0 \ \ldots\text{a}$$

Similarly

$$\Sigma(k\ O \cdot x) = \Sigma(y \cdot x) \text{ and}$$

$$\Sigma(y - kO)\ x = \Sigma(k\ a\ .\ x) = 0 \ \ldots\text{b}$$

In order that equations (a) and (b) shall hold good the line kk must be placed at such a distance above OO_1 that the ordinates k a measured above k k, then regarded

as positive, shall equal the ordinates k a measured below k k, then regarded as negative.

Let N = number of ordinates

e_1 = constant length of each ordinate of rectangle $0\ k\ k\ 0_1$

Σy = sum of ordinates of neutral axis segment $0\ a\ 0_1$ Then by the preceding,

$$Ne_1 = \Sigma(y), \text{ hence}$$

$$e_1 = \frac{\Sigma(y)}{N}$$

The value e_1 measured vertically from 0 and 0_1 determines the location of what may be termed the temperature norm k k. The function of k k in temperature stresses has already been discussed.

From Table IV we find that $\Sigma(y) = 162.90$; since $N = 20$

$$e_1 = \frac{162.90}{20} = 8.145$$

Now since $\Sigma(e_1) = Ne_1$, and $\Sigma(y) = Ne_1$ it follows that

$$\Sigma(y) - \Sigma(e_1) = 0$$

hence

$$\Sigma(y - e_1) = 0$$

therefore, since $(y - e_1) = k\ a$

$$\Sigma(ka) = 0$$

Furthermore on account of the symmetrical disposition of the y ordinates

$$\Sigma(k\ a\ .\ x) = 0$$

Hence for the value of $e_1 = \frac{\Sigma(y)}{N}$ we fulfill the conditions.

$$\Sigma(k\ x) = 0$$

$$\Sigma(k\ a) = 0$$

$$\Sigma(\ a\ c) = 0$$

This is seen to be true from Table VII.

(g) Development of Pressure Curve of Arch by superposition of Equilibrium Polygon on Arch.- (1) Conditional equation, $\Sigma(M\ y) = 0$ must now be satisfied. (2) Determination of True Pole-Distance H. (3) Construction of Pressure Curve.

~~(1)~~ We must, in compliance with the procedure outlined, regard the neutral axis itself as an equilibrium polygon subjected to some system of loading. The closing line $m\ m_1$ of the basic equilibrium polygon must then be made coincident with the closing line k k of the neutral axis polygon. Their areas will then partially cover each other, and the real bending moments acting in the arch will be proportional to the ordinates intercepted between these two polygons.

Both polygons, in reference to their own closing lines, satisfy the conditions.

$$\Sigma(M) = 0 \text{ and } \Sigma(M x) = 0.$$

There remains, to be satisfied, the conditions expressed by equation

$$\Sigma(My) = 0.$$

which is expressed by

$$\Sigma(a\ c\ .\ y) = 0.$$

Since the points along the basic polygon have been represented by the letter b and those along the neutral axis by the letter a, when the basic polygon is superposed on the arch the imaginary intercepted ordinates will be of the type b a.

Now the bending moments of the sections S of the arch must be proportional to the intercepted ordinates of type b a, hence

$$\Sigma(\ b\ a\ .\ y) = 0$$

Ordinates of basic polygon are of the type m b.

Ordinates of neutral axis polygon are of the type k a.

hence

$$mb - ka = ba$$

therefore

$$\Sigma(mb\ .y) - \Sigma(ka.y) = \Sigma(ba\ .y) = 0.$$

whence

$$\Sigma(mb\ ,y) = \Sigma(ka\ .y) = (c)$$

The two members of equation (c) must now be evaluated and if equality does not exist the ordinates of the type m b must all be changed in such a constant ratio that equality will ensue. In order to achieve this result, suppose that,

$$\frac{\Sigma(k\ a\ .\ y)}{\Sigma(m\ b\ .\ y)} = U$$

then

$$U\,\Sigma(m\ b\ .\ y) = \Sigma(k\ a\ .y)$$

But

$$k\ a = y - e_1$$

hence

$$\Sigma(k\ a\ .\ y) = \Sigma(y - e_1)\ y$$

therefore

$$\Sigma(k\ a\ .\ y) = \Sigma(y^2) - e_1\,\Sigma\ (y)$$

whence

$$U = \frac{\Sigma(k\ a\ .\ y)}{\Sigma(m\ b\ .\ y)} = \frac{\Sigma(y^2) - e_1\,\Sigma\,(y)}{\Sigma(m\ b\ .\ y)}$$

It follows that if the ordinates m b be multiplied by the ratio U we will obtain the ordinates k c of the pressure curve which will satisfy equation c, and hence make $\Sigma(M\ y) = 0$.

Let the points so determined, on the pressure curve, be designated by the letter c.

Therefore

$\Sigma (a\,c\,.\,y) = 0$, which is to be satisfied

Hence

$$k\,c = m\,b.\ \frac{\Sigma(y^2) - e_1\,\Sigma(y)}{\Sigma\,(m\,b\,.\,y)} \quad .....d$$

In order to solve equation d we must evalueate the terms $\Sigma(y^2) - e_1 \Sigma (y)$ and $\Sigma (\,m\,b\,.\,y)$. The values m b are found from the basic polygon by scaling, using the scale of distance,

From Table IV, we have

$\Sigma (y) = 162.90$

$\Sigma (y^2) = 1401.22$

$e_1 = 8.145$

Hence

$$\Sigma(y^2) - e_1 \Sigma(y) = 1401.22 - 8.145 (162.90 = 73.26$$

Since symmetrically disposed ordinates y of the neutral axis are equal, it follows that there will be two symmetrical values of m b which will be multiplied by the same value of y; in the evaluation of $\Sigma(m\ b\ .\ y)$. Thus

$$(m_1 b_1 \quad m_{20} b_{20})\ y1;\ (m_2 b_2 \quad m_{19})\ y_2, \text{ etc.}$$

The evaluation of $\Sigma(m\ b\ .\ y)$ is given in detail in Table VI, from which it is seen that

$$\Sigma(m\ b\ .\ y) = \quad 93.21$$

Care must be taken to observe that m b ordinates above m m_1 are regarded as positive, and below as negative.

It follows that,

$$\frac{\Sigma(y^2) - e_1 \Sigma(y)}{(m\ b\ .\ y)} = \frac{(73.26}{93.21} = 0.786$$

Hence equation (d) becomes

$$k\ c = mb.\ \frac{\Sigma(y^2) - e_1\ \Sigma(y)}{\Sigma(m\ b.\ y)} = m\ b\ 0.786$$

The values of k c are found in coloumn (3) of Table VII.

The points c_1, c_2, c_3 . . . . c_{19}, c_{20} of the pressure may now be plotted by scaling the values of k c from Table VII, laying them off from the line k k, along the respective ordinates through points a.

(2) We must now determine the value of the True Pole Distance H. Let O' be the trial pole and H_1 the trial pole distance. From O' draw the ray O'G parallel to the true closing line m m_1. Then the point of intersection G of the ray O'G with the load line A B determines absolutely the value of the two end reactions AG and BG. But the true closing line of the arch is the horizontal line k k, hence the ray for the true pole-distance must be parallel to k k, hence horizontal, and must start from the point of junction of the two end reactions, that is from point G.

The length of the true pole-distance, G O = H may be found from the following considerations:

Let A b B, whose ordinates are of the type m b, be the polygon corresponding to the trial ray diagram O' A B; and let A c B, whose ordinates are of the type kc, be the polygon corresponding to the ray diagram O A B; thus assuming that the true pole O has been found. Hence

A b is parallel to B O'

b B is parall to A O'

A c is parallel to B O

c B is parallel to A O

A B is parallel to G O and B O'

Therefore triangle A b B is similar to triangle O' A B, and triangle A c B is similar to triangle O A B.

Hence

$$\frac{H}{H_1} = \frac{mb}{kc} = \frac{\Sigma(m\ b\ .\ y)}{\Sigma(\ k\ c\ .\ y)}$$

Therefore

$$H = H_1 \ \frac{\Sigma(\ mb\ .\ y)}{\Sigma(\ k\ c\ .\ y)}$$

or

$$H = H_1 \ \frac{\Sigma(\ m\ b\ .\ y)}{\Sigma(y^2) - c_1 \Sigma(y)}$$

hence

$$H = 35,000 \ \frac{93.21}{13.26} = 44,529 \text{ lbs.}$$

If the value 44,529 be laid off from G to O along the horizontal line G O we locate the true pole O.

(3) We can now proceed to construct the Pressure Curve. Complete the true ray diagram by drawing the rays from the true pole O.

Through point C_{12} draw line A B between load lines P_{12} and P_{13} parallel to ray O B which meets the point of juncture B of loads P_{12} and P_{13} in the ray diagram. Similarly, through C_{13} draw B C parallel to O C of the ray diagram. If the calculations and graphical constructions have been carefully made lines A B and B C so drawn will intersect at B on the load line P_{13}.

By continuing this process the entire pressure curve can be drawn as shown.

Through point C_{20} a line is drawn between load lines P_{20} and P E (left) parallel to ray O B which intersects the load line in the point of juncture of load P_{20} and Left End load.

Where the line through C_{20} intersects the load P E (left) we draw the final and completing line of the pressure curve parallel to the ray OC.

18. Bending Moments, Thrusts and Shears.- The bending moment M at any point a of the arch is given by

$$\sum(M) = H\ (\ a\ c\ )$$

Hence we scale the ordinates k a and record them in column 4 (Table VII).

Then since

$k\ c - k\ a = a\ c$ we obtain the values from which the bending moments can be calculated as shown in detail in Table VIII.

To obtain the thrusts and shears at the various points a we construct tangents to the neutral axis. Now as the neutral axis is an arc of a circle we need only draw radii to the various points a and erect at a perpendiculars to the respective radii.

Then from the true pole O we draw a line parallel to this tangent corresponding to the ray which refers to the point in question. Thus for point a_{20}, the tangential thrust (tangential component) is found by drawing from point O of the ray diagram a line O S parallel to the tangent through a_{20} and constituting one side of the right triangle O S B whose hypotenuse O B is parallel to the pressure line through C_{20}. The remaining side B S of the right triangle is the normal component and is proportional to the shear for the particular point considered. By scaling these distances to the scale of loads we determine the amount of the thrust and shear for the various points a of the neutral axis.

Calculation of the Fiber Stresses for a Typical Point.

$$S_c = \frac{T}{A_c + n\ A_s} \pm \frac{M x_1}{\frac{1}{12} D^3 + n\ A_s\ x_2^2} \text{ (lbs. per Sq. Ft.}$$

Let

E_s = modulus of elasticity of steel = 30,000,000 lbs. per sq. in.

E_c = modulus of elasticity of concrete = 1,500,000 lbs. per sq. in.

Then

$$n = \frac{E_s}{E_c} = \frac{30,000,000}{1,500,000} = 20.$$

A_c = area section concrete in a slice 1 ft. wide and D ft. deep. (Value of D, Table II) =

$D \times 1 = D$ sq. ft.

A_s = area steel in square ft. in a slice 1 ft. wide $= \frac{A}{b}$;

where

A = cross-sectional area in square ft. of two bars constituting a rib.

and b = distance (in feet) c. to c. of ribs.

Area 7/8 in. corr. barr = 0.77 sq. in. =. 0.00534 sq.ft.

$A = 2\ (0.00534) = 0.01068.$

$b = 8$ ins. = 2/3 ft.

$$A_s = \frac{A}{b} = \frac{0.01068}{2/3} = 0.016 \text{ (a constant)}$$

$n\ A_s = 20\ (0.016) = 0.32$ (a constant)

D = radial depth in feet of arch ring at point a.

G = distance in feet from c.g. of steel bar to extreme fiber of concrete

= 3 ins. (assumed) = 0.25 ft.

D' = distance in feet between centers of gravity of two steel bars constituting a rib.

$= D - 2G$; $2G = 2\ (0.25) = 0.50$ (an assumed constant)

$= D - 0.50$ (in ft.)

$$x_1 = \frac{D}{2}$$

$$x_2 = \frac{D'}{2} = \frac{D - 0.50}{2}$$

T = Thrust in lbs. (Tangential component at point a).

For value see Table VIII.

M=Bending moment in foot pounds at point a.

= H (a c). For value see Table VIII.

Fiber Stress in Concrete.-

$$S_c = \frac{T}{A_c + n\ A_s} \pm \frac{M\ X_1}{\frac{1}{12}\ D^3 + n\ A_s\ X_2^2}$$

Fiber Stress in Steel.

$$S_s = \left(\frac{T}{A_c + n\ A_s} \pm \frac{M\ X_2}{\frac{D^3}{12} + n\ A_s\ X_2^2} \right) n$$

When a c is + then bending moment M is +

When a c is - then bending moment M is -

When M is + then + of ± refers to upper fiber of concrete and steel.

When M is + then - of ± refers to lower fiber of concrete and steel.

When M is - then + of ± refers to upper fiber of concrete and steel.

When M is - then - of ± refers to lower fiber of concrete and steel.

Calculation of Fiber Stresses Due to Variation in Temperature.

H_t = horixontal thrust due to change in temperature.

t^o = number of degrees change of temperature

= ± 60° F.

L = Span in feet.

E'_c = modulus of elasticity of concrete in lbs. per sq. ft.

= 144 E_c = 144 (1,500,00)

(E_c = modulus per sq. in.)

e = coefficient of expansion of concrete for 1° F. = 0.000006

I_c = moment of inertia of concrete of area A_c

I_s = moment of inertia of steel of area S_2.

$$\frac{I_c + nIS}{S} = \text{a constant.}$$

Calculate value of $\frac{I_c + nIs}{S}$ for 3 points at a_{20}, a_{15}, a_{11}, and use the average of these values. This will be accurate enough for all practical purposes.

For point a_{20}:-

$$\frac{I_c + nIs}{S} = \frac{1/12\ D^3 + 20\ (\frac{D}{2} - G)^2}{S} = \frac{\frac{(2.43)^3}{12} + 20 \times .016(121-.25^2}{13.40}$$

= .185

For point a_{15}:-

$$\frac{Ic + nIs}{S} = .114$$

For point a_{11}:-

$$\frac{I_c + nIs}{S} = .114$$

Therefore use at an approximate value the average of the above 3 values or $.138 = \frac{I_c + nIs}{S}$

$$H_t = \frac{El_c let^o}{2(\Sigma(y^2) - l_1 \Sigma(y)} \quad \frac{I_c + nIs}{S} = \frac{1,500,000\ (144)\ 72\ (.000006)\ 40\ (.138)}{2 \quad 73.5}$$

$$= 3505$$

Fiber Stresses Calculated - For Point 1.

Concrete

$$S_c = \frac{T}{A_c + nAs} \pm \frac{M\ x_1}{\frac{1}{12} D_1{}^3 + n\ As \quad X_2^2}$$

T = 50,200 (See Table VIII, point 1)

$A_c = 2.43$ (Table X)

$n\ A_s = 20\ (0.016) = 0.32$

$M = 11,400$ (Table VIII, point 1)

$X_1 = \frac{D}{2} = \frac{2.43}{2} = 1.21$ (Table X)

$D^1 = D - 2\ G = 2.43 - 2\ (0.25) = 1.93$

$X_2 = \frac{D^1}{2} = \frac{1.93}{2} = .96$

$X_2^2 = .92$

$D^3 = 14.34$

$$S_c = \frac{50,200}{2.43 + 0.32} \pm \frac{11,400 \times 1.21}{\frac{14.34}{12} + 0.32 \quad .92} = (18,240 \pm 9,330)$$

M is +

+ of ± gives + 23,570 lbs. per sq. ft. = per sq. ft. = +191 lbs. per sq. in. compression in the upper fiber of the concrete - of ± gives + 8,910 lbs. per sq. ft. = + 62 lbs. per sq. in. compression in lower fiber of concrete.

Steel.

$$S_s = \left(\frac{T}{A_c + n\ As} \pm \frac{Mx_2}{\frac{1}{12} D_1^3 + nAs\ \frac{2}{2}} \right) n$$

$$= \left(\frac{50,200}{(2.43 + 0.32} \pm \frac{11,400 \cdot .96}{\frac{14.34}{12} + .32 \cdot .92} \right) 20$$

$$= (+ 18,240 \pm 7,380)\ 20$$

M is +

+ if ± gives + 512,400 lbs. per sq. ft. = + 3,550 lbs. per sq. in. compression in upper fiber of steel.

- if ± gives + 217,200 lbs. per sq. ft. = + 1,505 lbs. per sq. in compression in lower fiber of steel.

Temperature Concrete.

$$S_{ct} = \frac{T_t}{A_{ct} + nAs} \pm \frac{Mt\ X_1}{\frac{1}{12}\ D_1{}^3 + n\ As \times \frac{2}{2}}$$

$$M_t = H_t\ (ka) = -\ 17,840 \quad \text{(Table X)}$$

$$T_t = H_t\ \cos \quad = 3,175 \quad \text{(Table IX)}$$

$$S_{ct} = \frac{3,175}{(2.43 + 0.32)} \pm \frac{-17,840\ \ 1.21}{\frac{14.34}{12} + 0.32\ \ .92}$$

+ if gives - 93 lbs. per sq. in.

- if gives +109 lbs. per sq. in.

Temperature Steel.

$$S_{st} = \left(\frac{3,175}{2.43 + 0.32} \pm \frac{17,840 \quad .96}{\frac{14.34}{12} + .32 \qquad .92}\right) 20$$

+ if ± gives - 1,430 lbs. per sq. in.

- if ± gives + 1,760 lbs. per sq. in.

Design of the Abutment. (Plate 4).

In this diagram the various forces acting upon the abutment are shown in magnitude, direction and point of application for a vertical, longitudinal rib or strip, one foot in thickness. The thrust of the arch is denoted by P_1. P_2 and P_3 are the weights of the two trapezoidal prisms of filling and concrete; P_4 represents the lateral pressure of the earth filling acting on the vertical projection of the abutment and tending to force it forward into the stream; P_5 is the pressure of the water acting on the face of the abutment. These various forces when combined produce a resultant action, R, on the base of the abutment having a magnitude of 101,000 pounds and passing within the middle third of the base, as it should in a properly designed abutment.

GENERAL SPECIFICATIONS FOR CONCRETE-STEEL STRUCTURES.

1. PLANS.- The work will be constructed in accordance with the drawings, herewith submitted, and these specifications.

The specifications and drawings are intended to describe and provide for the complete work. They are intended to be co-operative, and what is called for by either is as binding as if called for by both.

The work herein described is to be completed in every detail, notwithstanding that every item necessarily involved is not particularly mentioned.

The contract price shall be based upon these specifications and drawings, which are hereby made a part of the contract.

2 Dead load concrete shall be assumed at 150 lbs. per cu. ft. Earth at 120 lbs. per cu. ft. Live load shall be computed on a basis of 200 lbs. per sq. ft. of roadway and sidewalk.

3. CONDITIONS OF CALCULATIONS.-

Modulus of elasticity of concrete 1,500,000 lbs.
Modulus of elasticity of steel 30,000,000 lbs.

MAXIMUM COMPRESSION ALLOWED ON CONCRETE.

In arches for highway and electric car bridges.

Exclusive of temperature stresses 375 lbs. per sq.in

Including stresses due to 40° variation in temperature 500 " " " "

MAXIMUM SHEAR ALLOWED ON CONCRETE 75 lbs. per sq.in.

MAXIMUM STRESS ALDOWED ON STEEL.

In arches. The steel ribs under a stress not ex- 18,000 lbs. per sq. in. must be capable of taking the entire bending moment of the arch without aid from the concrete, and have flange areas of not less than the one hundred and fiftieth part of the total area of the arch at crown. The actual stress when imbedded in and acting in combination with concrete shall not exceed twenty times the allowed stesss on the concrete.

4. DISCREPANCIES.- In the event of any discrepancies between the drawings and the figures written on them, the figures are to be taken as correct, and in case of any discrepancy between the drawings and the specifications are to be adhered to.

5. FOUNDATIONS.-All foundations shall be as shown on plans, and conform to the dimensions marked thereon.

Foundations on rock shall be prepared by removing all sand, mud, or other soft material, and by excavating the bed rock in such manner as may be described or shown on drawings.

Foundations on hard pan, gravel, gravel and clay, cemented sand, or other material intended to carry the load without piles, shall be excavated to the depths shown on plans.

Foundations on piles will usually be inclosed by permanent water-tight tongue and grooved sheet piling, or Wakefield piling, and be excavated to the depth shown on plans, and the piles shall be driven after the excatations are made. They shall be sawed off at least 1 ft. below low water. The spaces between the piles shall be filled with concrete, and in case it is found necessary to lay the concrete under water, proper applicnces must be used to insure its being deposited with as little injury as possible.

The piles shall be oak, yellow pine, or other wood that will stand the blow of the hammer; straight, sound, and cut from live timber; trimmed close, cut off square at the butt, and have all bark taken off. The piles shall not be less than 12 ins. nor more than 16 ins.

in diameter at the large end nor less than 8 ins. in diameter at the small end for piles having a length of 30 ft. and under. For greater lengths the diameter of the small end may be reduced 1 in. for each 10 ft. of additional length down to a minimum of 6 ins. The piles shall be driven until they do not penetrate more than 1/2 in. under the blow of a hammer weighing 2,240 lbs. falling 25 ft.

6. CEMENT. The cement shall conform to the specifications formed on page 75 of Brown's "Hand Book for Cement Users".

The sand used in preparing test specimens shall be clean, sharp, crushed quartz retained on a sieve of 30 meshes per lineal inch, and passing through a sieve of 20 meshes per lineal inch. In test specimens of 1 cement and 3 sand, no more than 12 per cent of water by weight shall be used. Specimens prepared from a mixture of 1 part cement and 3 parts sand, parts by weight, shall after seven days develop a tensile strength of not less than 170 lbs. per sq. in., and not less than 240 lbs. per sq. in. after 28 days. Cement mixed neat with from 20 per cent to 25 per cent of water to form a stiff paste shall after 30 minutes

be appreciably indented by the end of a wire 1/12 in. in diameter loaded to weight 1/4 lb. Cement made into thin pats on glass plates shall not crack, scale, nor warp under the following treatment: Three pats will be made and allowed to harden in moist air at from 60° to 70° Fahrenheit; one of these will be placed in fresh water for 28 days, another will be placed in water which will be raised to the boiling point for six hours and then allowed to cool, and the third is to be kept in air of the prevailing outdoor temperature.

7. PORTLAND CEMENT CONCRETE.- The concrete shall be composed of cement, sand, and broken stone or gravel mixed with clean water in the proportions hereafter mentioned.

The sand shall be clean, sharp, and coarse, or coarse and fine mixed, free from sewage, mud, clay, and all foreign matter.

The broken stone shall be clean and hard, broken into approximately cubical pieces, and free from long, thin scales.

The gravel shall be of assorted sizes screened or washed entirely free from clay, loam, or foreign matter, and be free from scale, slime, or humus.

Whenever the amount of work to be done is sufficient to justify it, and for all work exceeding 1,000 cu. yds., approved mixing machines shall be used. The ingredients shall be placed in the machine in a dry state, and in the volumes specified, and be thoroughly mixed, after which clean water shall be added and the mixing continued until the wet mixture is thorough and the mass uniform. The mixture shall be sufficiently wet for the water to come to the surface with moderate ramming. As soon as the batch is mixed it must be deposited in the work without delay. For small bridges, if the mixing is done by hand, the cement and sand shall first be thoroughly mixed dry, in the proportions specified. The stone, previously drenched with water, shall then be deposited in this mixture. Clean water shall be added and the mass be thoroughly mixed and turned over until each stone is covered with mortar, and the batch be deposited without delay.

The concrete shall be deposited in layers of 6 or 8 ins., and be thoroughly rammed until all voids are filled and the water flushes to the surface.

The grades of concrete to be used are as follows:

(a) For the arches, slabs, girders, floors, walls subject to transverse stress, posts and tanks, 1 part Portland cement, 2 parts sand, and 4 parts broken stone that will pass in any direction through a 1-1/2 in. ring, if not otherwise marked on plans.

(b) For spandrel walls, 1 part Portland cement, 3 parts sand, and 6 parts broken stone or gravel that will pass through a 2-in. ring.

(c) For the piers, abutments, foundations, and retaining walls, 1 part Portland cement, 3 parts sand, and 6 parts broken stone or gravel that will pass through a 3 in. ring.

8. ARTIFICIAL STONE.- (a) All keystones, brackets, consoles, dentiles, pedestals, parapets, hand railings, posts and panels, and other ornamental work when used; also curbs and gutters, shall be of the design shown on plans, and be molded in smooth and suitable molds. For moldings containing curved surfaces, sharp curves, carvings, or other delicate work, the molds shall be plastered with a semi-liquid mortar composed of 1 part cement and 2 parts offine sharp sand. The mortar coating must be followed up with a backing of only earth damp concrete composed of 1 part cement, 2 parts sand, and 4 parts of fine broken stone, or 1 part

cement and 6 parts of gravel that will pass through a 3/4 in. ring. The concrete backing must be rammed thoroughly in thin layers.

(b) For plain flat surfaces, the concrete may be rammed directly against the molds, and after the molds have been removed all exposed surfaces shall be floated to a smooth finish with a mortar the same as specified in 8 (a), care being taken that no body of mortar is left on the face, sufficient only being used to fill the pores and give a smooth finish.

When pedestal posts carry lamp posts, a 4 in. wrought iron pipe shall be built into the concrete from top to bottom, and at bottom it shall be connected with a 3 in. pipe extending under the sidewalk, and connected with gas pipe or electric wire conduit. The pipes shall have no sharp bends, all changes in direction being made by gentle curves.

9. PLASTERING.- No plastering will be allowed on the exposed faces of the work, but the inside faces of the spandrel walls covered by the fill shall be plastered with mortar composed of 1 part cement and 2-1/2 parts sand, the surface being well dampened before plastering.

10. MIXTURES.- The volumes of cement, sand broken stone, or gravel in all mixtures of mortar or concrete shall be measured loose.

11. CONNECTIONS.- In connecting concrete already set with new concrete, the surface shall be cleaned and roughened, and mopped with a mortar composed of 1 part cement and 1 part sand to cement the parts together.

12. EXPANSION JOINTS.- Expansion joints shall be made in the spandrel walls, cornices, and parapets of each arch above the springing lines, at points one-sixth span from the springing lines, and at such other points, if any, as are shown on plan.

13. SPANDRELS.- The spandrel walls shall have a thickness of not less than 18 ins. at any point, and a thickness at bottom of not less than four-tenths of the height of the wall measured from the top of cornice.

14. ARCHES.- For square arches the concrete shall be laid in transverse sections of the full width of the arch, between timber forms normal to the center line of the arch, the length of sections being such that the center section, or a pair of intermediate or end sections, shall constitute a day's work.

Work shall be started at the center section and carried towards the ends, the end sections being laid last.

15. DRAINAGE.- Provision for drainage shall be made at each pier as follows: A wrought iron pipe of sufficient diameter shall be built into the concrete, extending from the center of each space over piers to the soffit of the arch near the springing line, and project 1 in. below the soffit. The surface of the concrete over piers shall be so formed that any water that may seep through the fill above will be drained to the pipes. The line of drainage will be covered with a layer of broken stone, and the top of pipes will be provided with screens to prevent clogging.

16. STEEL.- Steel ribs shall be imbedded in the concrete of the arches. They shall be spaced at equal distances apart. The design, location, dimensions, and connections of the ribs, also the sections of steel of which they are composed, shall be as shown on plans.

Steel rods shall be imbedded near the tension side of all members subjected to transverse stress. All ends of rods shall anchored or bent up to develop full tensile strength. The distance of the center of the rods from the outside of the concrete shall not be less than the diameter of the rods.

All steel must free from paint and oil, and all scale and rust must be removed before imbedding in the concuret.

The tensile strength, limit of elasticity and ductility shall be determined from a test piece cut from the finished material and turned and planed parallel. The area of cross section shall not be less than 1/2 sq. in.; the elongation shall be measured after breaking on an original length of 8 ins. Each melt shall be tested for tension and bending.

Either soft or medium steel may be used in all concrete steel structures. If soft steel is used it shall have an ultimate strength of from 54,000 to 62,000 lbs. per sq. in., an elastic limit of not less than one-half the ultimate strength, shall elongate not less than 25 per cent in 8 ins., and bend cold 180° flat on itself without fracture on outside of bend. If medium steel is used it shall have an ultimate strength of from 60,000 to 68,000 lbs. per sq. in., an elastic limit of not less than one-half the ultimate strength, shall elongate not less than 22 per cent in 8 ins., and bend cold 180° to a diameter equal to the thickness of the piece tested without fracture on outside of bend.

In tension tests the fracture must be entirely silky. The workmanship must be first class.

17. CASING.- When concrete facing is used, all piers, abutments and spandrel walls shall be built in timber forms. These forms shall be substantial and unyielding, of the proper dimensions for the work intended, and all parts in contact with exposed faces of concrete shall be finished to a perfectly smooth surface by plastering or other means, so that no mark or imperfection shall be left on the work.

18. CONCRETE FACING.- If concrete facing is used, the concrete shall be deposited in smooth molds as specified in paragraph 17, and after the molds have been removed the exposed flat surfaces shall be finished in the same manner as specified in paragraph 8 (b).

It the arch faces, quions, or other exposed surfaces are marked to represent masonry or other division lines, either straight or curved, are shown in the faces of the arch or spandrels, such division lines shall be made by triangular moldings of wood 2 ins. wide and 1 in. deep, fastened to the casing in true lines as shown on plans. The face of the arch at intrados shall be beveled to correspond, and all angles or intersections of the moldings shall be neatly beveled and fitted in a workmanlike manner to give a smooth finish.

Before depositing the concrete the moldings shall be coated in the same manner as specified in paragraph 8 (a).

The soffits of the arches shall be floated and finished in the same manner as specified in paragraph 8 (b).

20. CENTERING.- The contractor shall build an unyielding false-work or centering. The lagging shall be dressed to a uniform thickness so that when laid it shall present a smooth surface, or it shall be made smooth by plastering or other efficient means.

In framing the centers allowance shall be made for settlement of centerings, deflection of arch after the removal of centerings and for permanent cambre. The centers shall be framed for a rise of arch greater than the rise marked on drawings by an amount equal to one-eight hundredth part of the span, and shall not be struck until at least 28 days after the completion of the arch, and not until the fill has been put on. Great care shall be used in lowering the centers evenly and uniformly, preferably by means of sand boxes, so as not to throw undue strains upon the arches. The tendency of the centers to rise at the crown as they are loaded at the haunches must be provided for in the design, or, if not, the centers must be temporarily loaded at the crown and the load so

regulated as to prevent distortion of the arch as the work progresses.

21. WATER-PROOFING.- After the completion of the arches and spandrels, and before any fill is put in, the top surface of the arches, piers and abutments, and the lower 6 ins. of the inner surface of the spandrel wall shall be coated with a heavy coat of semi-liquid mortar consisting of 1 part cement, 1/2 part thoroughly slaked lime, and 3 parts sand, spread to leave a smooth finish, and after this has set hard it shall be given a heavy coat of pure cement grout.

22. FILL.- The space between the spandrel walls shall be filled with sand, earth, cinders, or other suitable material, thoroughly compacted by ramming or rolling, and be finished to the proper grade to receive the curbing and pavement.

The fill over any arch shall not put in until at least two weeks after the arch concrete has been completed.

23. ROADWAY PAVEMENT.- The pavement shall be of the kind shown on plans, or mentioned in the proposal, and shall be built according to the specifications adopted in the locality where used unless otherwise mentioned.

24. **CONCRETING** SIDEWALKS.- The ground on which the concrete sidewalk is to be laid shall be rammed or rolled to a hard bearing surface, 10 ins. below the finished grade. After the curbing has been set true to line and grade, a foundation of gravel or cinders shall be laid and thoroughly rammed or rolled to a thickness of 6 ins. On this shall be deposited a layer of dry concrete, 3 ins. thick after ramming, consisting of 1 part Portland cement, 2 parts sand, and 4 parts broken stone or gravel that will pass through a 1-1/2 in. ring. On the concrete shall then be laid a wearing surface 1 in. thick, composed of 1 part Portland cement and 1-1/2 parts of coarse sharp sand or of broken granite or other acceptable stone in sixe from 1/8 in. downward. The mortar for the wearing surface shall not be too wet; it must be spread before the concrete base has had time to partiall set, shall be pressed down hard into the latter, and shall be ttoweled to a smooth and even surface. All concrete sidewalks shall be divided into blocks of not more than 36 sq. ft. All blocks shall be separated from those adjoining by pieces of heavy tar paper or other effective means to prevent adhesion of the blocks. The divisions between blocks shall reach entirely through

the concrete and the wearing surface, and shall be neatly finished on top with a jointing tool. As soon as the wearing surface has well set a 2 in. layer of sand shall be carefully spread over it and kept moist for on week by frequent sprinkling.

Concrete curbs shall be divided into blocks corresponding with those forming the sidewalk, and shall be neatly finished in the same manner.

25. HAND RAILING AND PARAPETS.- The hand railing or parapets shall be of the material and of the form and dimensions shown on plans, and shall be brought true to line, and be firmly fastened in the position shown. If an iron hand railing is used, it shall receive, after erection, two coats of paint of a color and quality approved by the engineer. Concrete pappets shall be made as specified in paragraph 8 (a). They shall be provided with expansion joints at intervals not greater than 10 ft.

26. LAMP POSTS AND TROLLEY POLES.- If required, and furnished by the contractor, they shall be of a decign approved by the engineer. The number of pieces and the minimum cost of each, delivered on the work, shall be specially mentioned.

27. NAME PLATES.- Two name plates shall be furnished by the contractor. They shall be of a design approved by the engineer, and built into the roadway side of the abutment pedestals or such other places as may be directed by the engineer, one plate being insceibed with the names of the city or county officials and year of completion, the other being inscribed with the names of the designers and contractors, and date of patents. The plates shall be made of bronze.

28. ERECTION.- The contractor shall employ suitable and competent labor for every kind of work. The contractor shall furnish all staging, piling, cribbing, centering, casing, and material of every description required in the erection of the work; also all plant, including dredges, engines, pumps, barges, pile drivers, derricks, mixing machines, conveyors, or other appliances necessary for carrying on all parts of work. The contractor shall make all the provisions necessary to maintain and protect buildings, fences, trees, conduits, sewers, and other structures, and shall repair and shall repair all damage occasioned; shall provide watchmen, red lights,fences, and other precautionary measures necessary to the protection of persons and property.

The contractor shall assume all risks for loss or damage incurred by ice, floods, fire, or other causes during the construction of the work, and until the same is accepted.

29. WORK EMBRACED BY CONTRACT.- The contractor shall do all the work prescribed in these specifications and as shown on the plans, for the structure complete from out to out of abutments or retaining walls, including fill, pavement, curbs, sidewalks, and hand railing or parapets for this length, unless otherwise mentioned.

30. APPROACHES.- The approaches will commence where the work mentioned in paragraph 29 ends, and they are not included in the contract, except when specially mentioned.

31. CLEANING UP.- After the completion of the work, and before final acceptance thereof, the contractor shall remove all temporary structures and rubbish, and leave the work and surrounding grounds in a neat and satisfactory condition.

32. MAINTAINING PUBLIC TRAVEL. - If public travel is to be maintained during the construction of the new bridge, by the constrcution of a temporary bridge, or otherwise, it shall be specially mentioned.

33. REMOVAL OF OLD BRIDGE.- If the site of the proposed structure is occupied by an old bridge, the same shall be removed by the contractor. The iron work shall be piled on the bank, and the timber and stone shall become the property of the contractor.

34. ENGINEER.- Whenever the work "Engineer" is used in these specifications, the same shall mean the Engineer or other public official who may be properly authorized to act for the party who makes this contract with the contractor.

35. LINES AND GRADES.- Lines and grades will be established by the Engineer, and no work shall be commenced until these are given.

36. EXTRA WORK.- The contractor must be prepared to do any extra work that may be ordered in writing by the Engineer, and for this he shall be paid at current contract rates for work of a similar character, or if the extra work should be of a class for which no rate is fixed by current contracts, the actual reasonable cost to the contractor, as determined by the Engineer, plus 15 per cent of said cost. The contractor shall have no claim for compensation for extra work unless the same is ordered in writing by the Engineer.

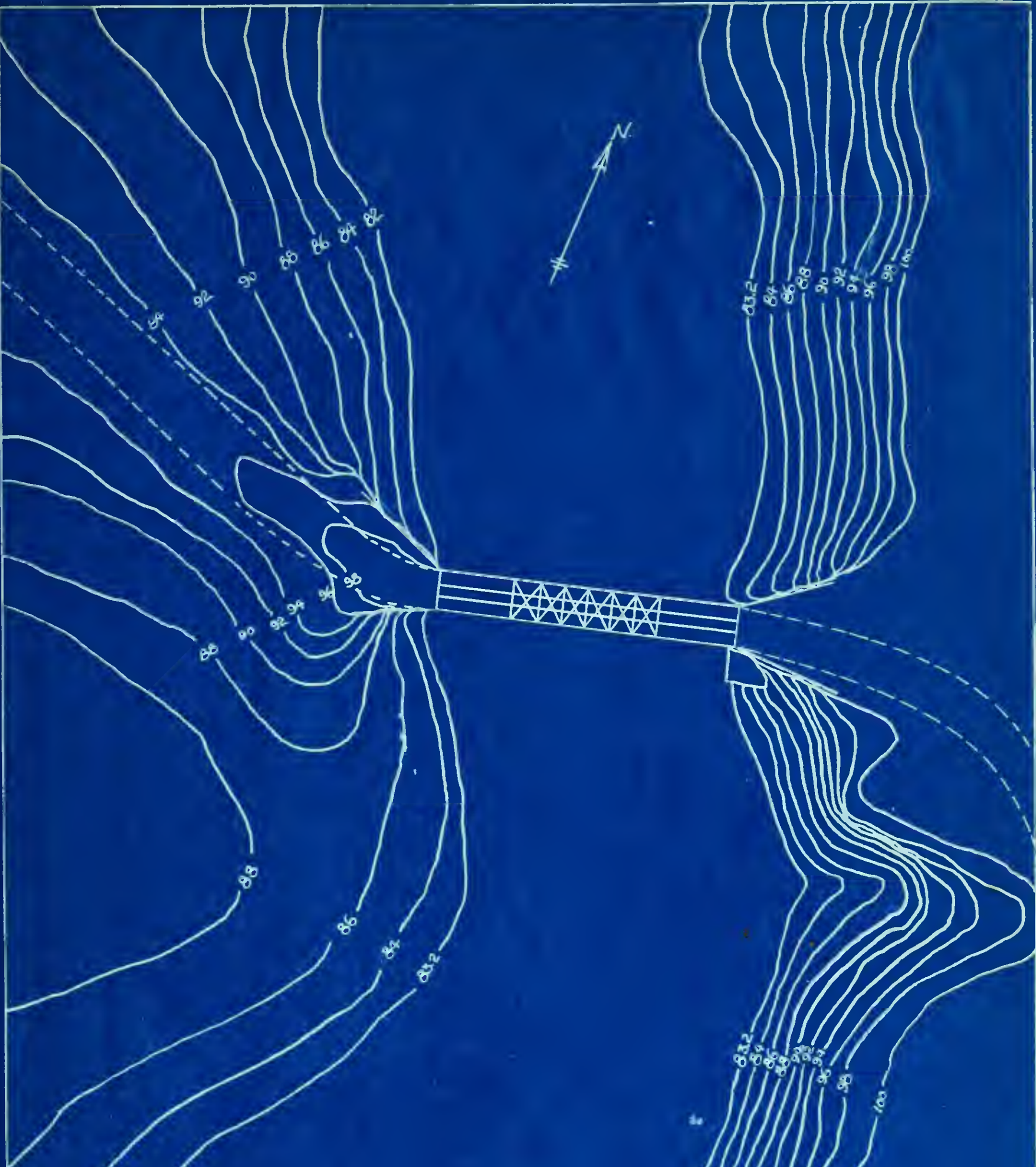
37. INSPECTION.- All material furnished by the contractor shall be subject to the inspection and approval of the Engineer, and the Engineer shall have power to condemn all work which in his opinion is not done in accordance with this contract and specifications.

38. MODIFICATIONS.- Any modifications of the prescribed lines, grades, positions, methods, or materials of construction which in the judgement of the Engineer may be expedient, shall be made by the contractor.

39. ESTIMATES.- Approximate estimates of the work done and material furnished shall be made on or about the last day of every month, and a valuation of the same in proportion to contract prices for the completed work will be made by the Engineer, which sum will be paid to the contractor in cash on or about the 10th day of the following month, less a deduction of 10 per cent upon said valuation, which shall be retained until the final completion of the work.

40. INTERPRETATION OF PLANS AND SPECIFICATIONS. The decision of the Engineer shall control as to the interpretation of the plans and specifications during the execution of the work thereunder, but this shall not deprive the contractor of his lawful rights to redress after the completion of the work for any improper orders or decisions which may have been received during execution of the work.

41. FINAL PAYMENT.- Upon the completion of the work, the contractor shall be promptly paid the balance of the contract price which shall remain due and unpaid.



TOPOGRAPHIC MAP OF ARCH SITE

PLATE I. R. P. Pearce — R. E. Guthrie. Scale:- 1" = 60'

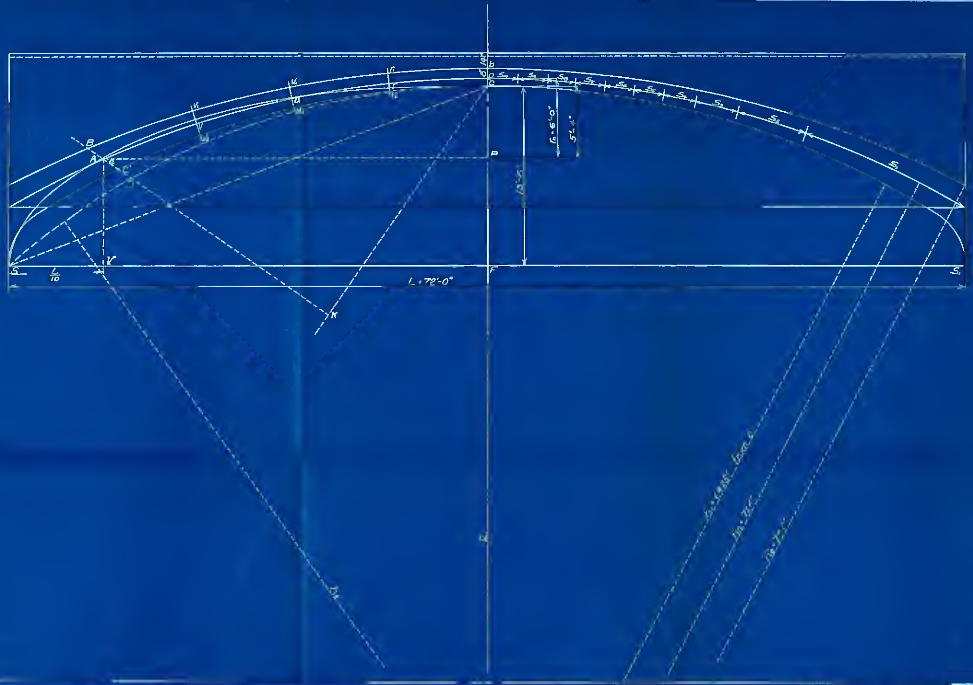


PLATE 2

CONSTRUCTION FOR CONCRETE ARCH
SHILLER PARK, ILLINOIS

Scale: 1"=4'
R. W. Guthrie

PLATE 3

DESIGN OF
REINFORCED CONCRETE ARCH

SHILLER PARK, ILLINOIS

Scales { Loads:- 1" = 5000 #
Arch :- 1" = 4'

MAY, 1910.

R. P. Pearce — R. E. Gauthier.

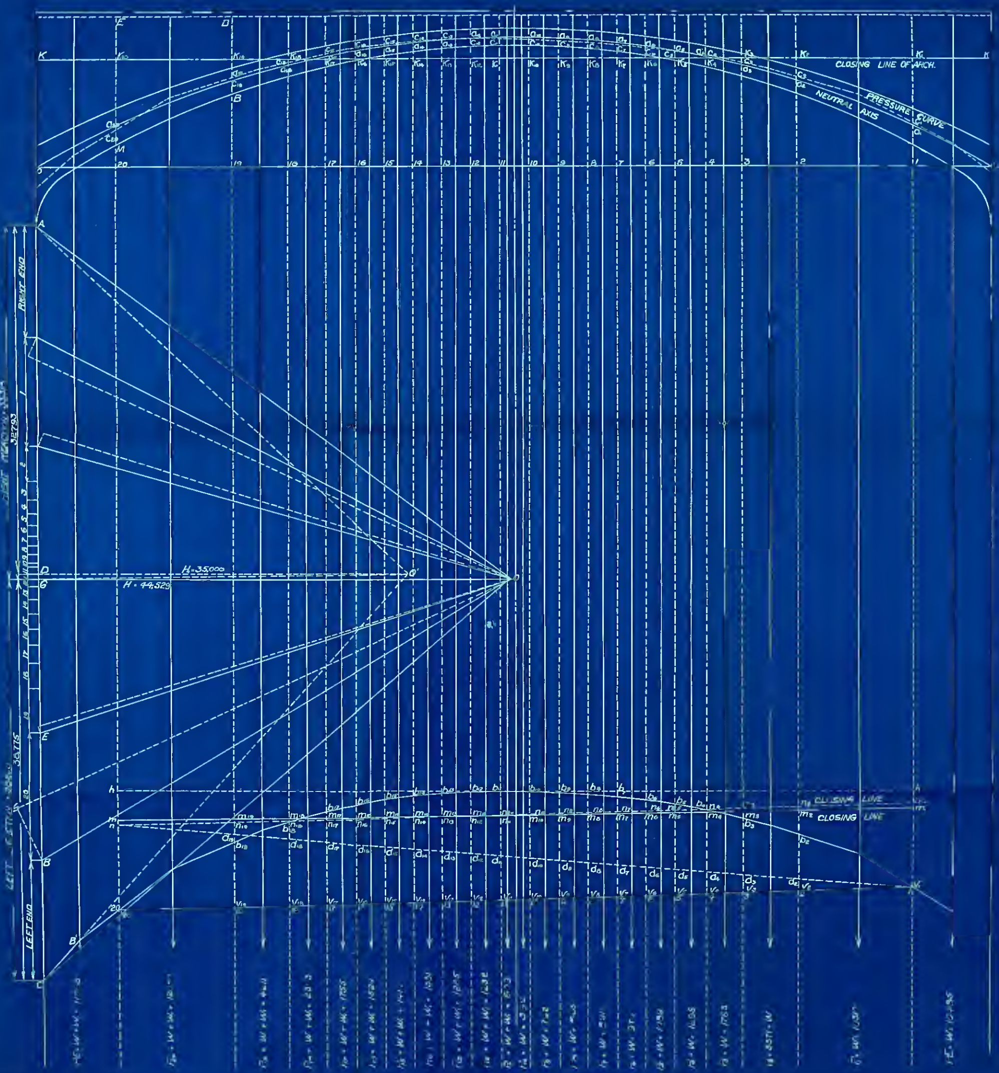


PLATE 3

DESIGN OF

REINFORCED CONCRETE ARCH

SHILLER PARK, ILLINOIS

Scales

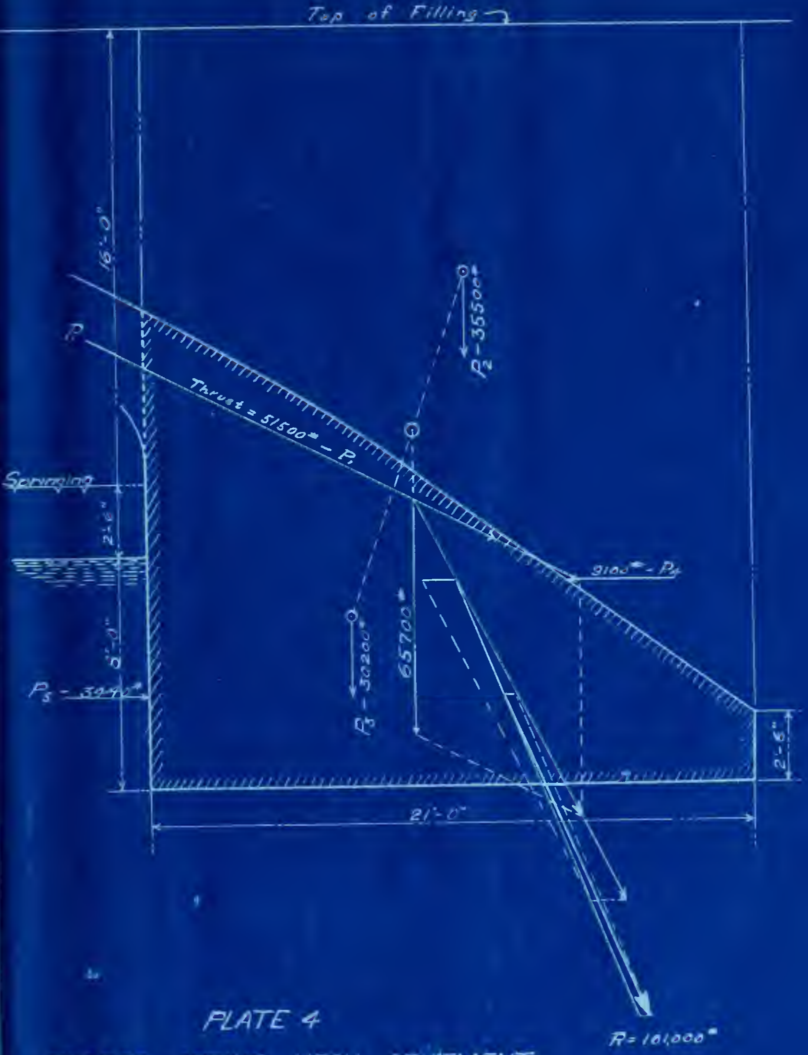
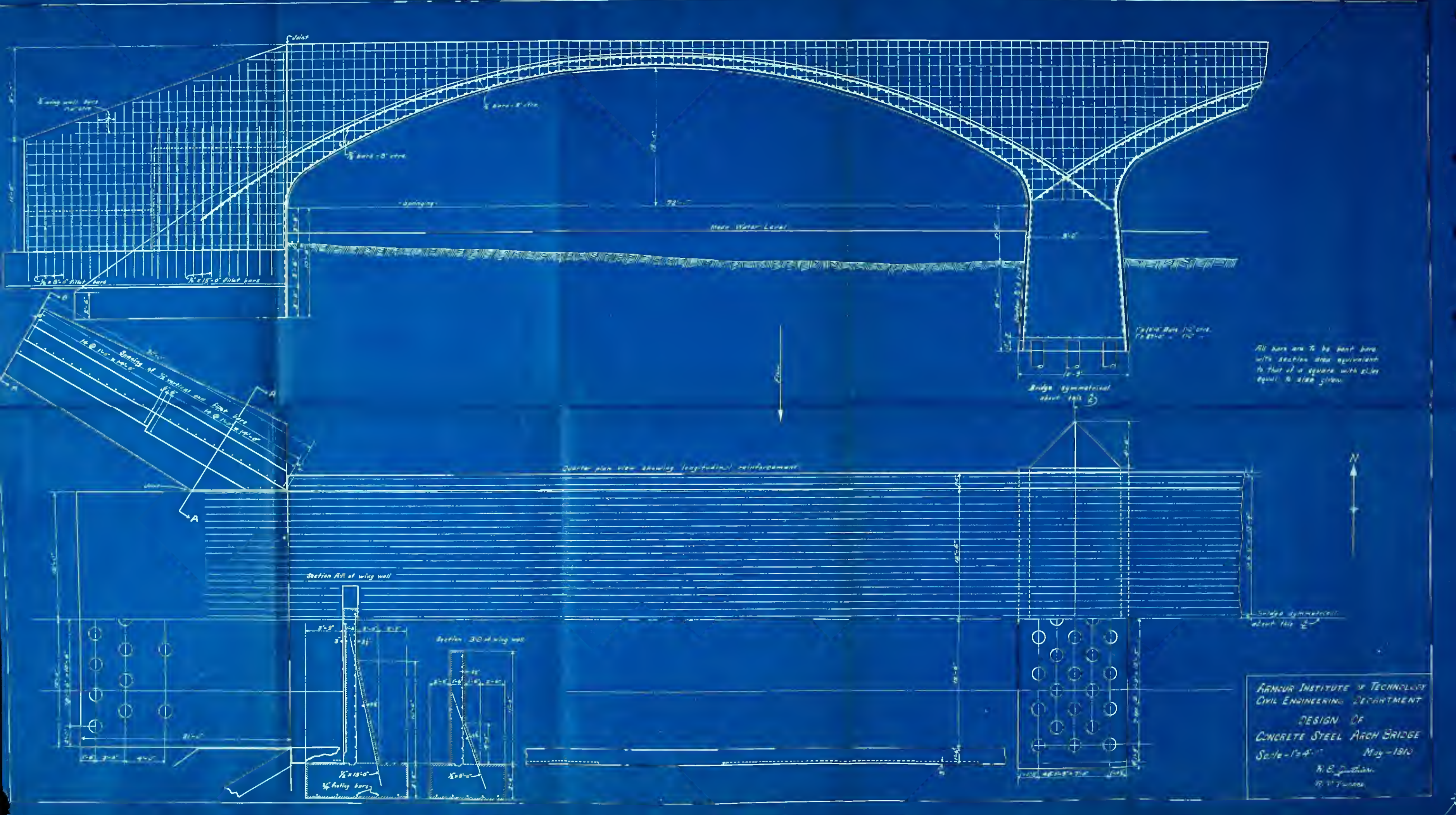


PLATE 4

FORCES ACTING UPON ABUTMENT

Scales { Outline:- 1" = 5'
Forces:- 1" = 4000#

R. E. Guthier.
R. P. Peake.


Joint
½" wing wall bars 1'-0" ctrs.
½" x 8'-0" fillet bars
¾" x 15'-0" fillet bars
¾" bars - 8" ctrs.
¾" bars - 8" ctrs.
Springing
72'-0"
Mean Water Level
8'-0"
1" x 12'-0" Bars 1'-0" ctrs.
1" x 27'-0" " 1'-0" "
12'-9"
Bridge symmetrical about this ℄
Spacing of ½" vertical and fillet bars
14 @ 1'-0" = 14'-0"
14 @ 1'-0" = 14'-0"
A
A
B
B
Joint
Quarter plan view showing longitudinal reinforcement
N
Section A-A of wing wall
Section B-B of wing wall
¾" x 15'-0"
¾" footing bars
¾" x 8'-0"
Bridge symmetrical about this ℄
All bars are to be bent bars with section area equivalent to that of a square with sides equal to size given.

ARMOUR INSTITUTE OF TECHNOLOGY
CIVIL ENGINEERING DEPARTMENT

DESIGN OF
CONCRETE STEEL ARCH BRIDGE
Scale-1"=4'-0" May-1910
R. E. Guthrie
R. V. Turner

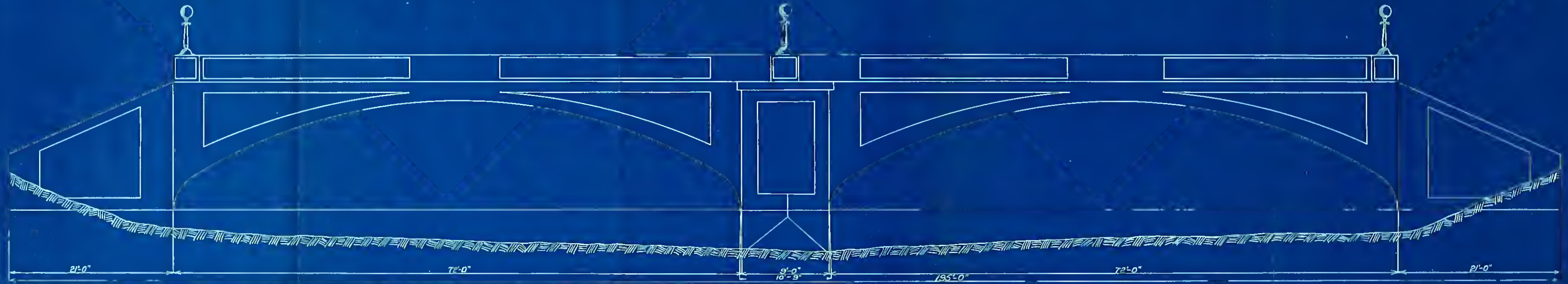


PLATE 6

ELEVATION OF CONCRETE ARCH

SHILLER PARK, ILLINOIS

Scale – 1"=8' MAY, 1910

H. F. [illegible] R. E. [illegible]

TABLE I. SUBDIVISION OF NEUTRAL AXIS

Constant	.768		1.153		.940		.881		.934	
Division	First Trial		Second Trial		Third Trial		Fourth Trial		Final Trial	
	Length	Sum'tion	L	S	L	S	L	S	L	S
S_{20}	12.00		15.00		13.51		13.30		13.40	
19	4.94	16.94	5.38	20.38	5.29	18.80	5.04	18.34	5.35	18.75
18	3.26	20.20	3.44	23.82	3.46	22.26	3.28	21.62	3.34	22.09
17	2.47	22.67	2.84	26.66	2.40	24.66	2.46	24.08	2.46	24.55
16	1.94	24.61	2.72		2.26	26.92	2.11	26.19	2.24	26.79
15	1.84	26.36	2.72		2.22		2.07		2.20	
14	1.81		2.72		2.22		2.07		2.20	
13	1.81		2.72		2.22		2.07		2.20	
12	1.81		2.72		2.22		2.07		2.20	
11	1.81	33.59	2.72	42.98	2.22	38.02	2.07	36.54	2.20	37.79
	33.59		42.98		38.02		36.54		37.79	

TABLE 2

Distance Along Neutral Axis from Springing Line in Feet	Depth of Arch Ring in Feet.	Distance	Depth	Distance	Depth
1	2.95	14	1.90	27	1.33
2	2.85	15	1.82	28	"
3	2.80	16	1.80	29	"
4	2.70	17	1.70	30	"
5	2.60	18	1.65	31	"
6	2.50	19	1.60	32	"
7	2.40	20	1.55	33	"
8	2.30	21	1.50	34	"
9	2.20	22	1.45	35	"
10	2.15	23	1.40	36	"
11	2.10	24	1.35	37	"
12	2.05	25	1.35	37.8	"
13	2.00	26	1.33		

TABLE III. CALCULATION OF LOADS P

	1			2	3	4	5	6	7	
	Trapezoid			Altitude	Vol. Cu. Ft.	D.L.	Length of	Area Sec.	L.L.	Total D.L.+L.L.
Division	Long Side (a)	Short Side (b)	½(a+b)	Trapezoid (c)	Sec. 1 Ft. Thick	150#/cu. ft. (Col. 3 X 150)	Division (c)	1' Wide Sq. Ft.	200#/sq. ft. (Col. 6 X 200)	(Col. 4 + Col. 7)
PE Right	13.5	9.8	11.65	6.0	69.90	10,485	6.0	6.0		10,485
P_1	9.8	6.0	7.90	8.7	68.73	10,310	8.7	8.7		10,310
2	6.0	4.7	5.35	4.2	22.47	3,371	4.2	4.2		3,371
3	4.7	4.0	4.35	2.7	11.75	1,763	2.7	2.7		1,763
4	4.0	3.5	3.75	2.3	8.63	1,295	2.3	2.3		1,295
5	3.5	3.1	3.30	2.2	7.26	1,089	2.2	2.2		1,089
6	3.1	2.8	2.95	"	6.49	974	"	"		974
7	2.8	2.6	2.70	"	5.94	891	"	"		891
8	2.6	2.4	2.50	"	5.50	825	"	"		825
9	2.4	2.4	2.40	"	5.28	792	"	"		792
10	2.4	2.3	2.35	1.1	2.59	389	1.1	1.1		389
11	2.4	2.3	2.35	1.1	2.59	389	1.1	1.1	2.20	609
12	2.4	2.4	2.40	2.2	5.28	792	2.2	2.2	4.40	1,232
13	2.6	2.4	2.50	"	5.50	825	"	"	"	1,265
14	2.8	2.6	2.70	"	5.94	891	"	"	"	1,331
15	3.1	2.8	2.95	"	6.49	974	"	"	"	1,414
16	3.5	3.1	3.30	"	7.26	1,089	"	"	"	1,529
17	4.0	3.5	3.75	2.3	8.63	1,295	2.3	2.3	4.60	1,755
18	4.7	4.0	4.35	2.7	11.75	1,763	2.7	2.7	5.40	2,303
19	6.0	4.7	5.35	4.2	22.47	3,371	4.2	4.2	8.40	4,211
P_{20}	9.8	6.0	7.90	8.7	68.73	10,310	8.7	8.7	17.40	12,050
PE Left	13.5	9.8	11.65	6.0	69.90	10,485	6.0	6.0	12.00	11,685
								Total D.L. + L.L. in lbs. =		71,568

TABLE IV. CALCULATIONS FOR LOCATION OF RESULTANT R.

Points on Neutral Line	1	2	3	4	Equilibrium Polygon	
	y Ordinates to Neutral Axis of Arch	y^2	vb Ordinates from Line $V_{20}V_1$ to Eq. Polygon	vb	Z Lever Arm About Center Line AB	Moments About Center Line AB
a_1	$y_1 = 3.15$	9.92	$V_{20}b_{20} = 0.00$	$V_1b_1 = 0.00$	$Z_1 = 30.10$	$(V_{20}b_{20} - V_1b_1)Z_1 = 0.00$
a_2	$y_2 = 6.45$	41.60	$V_{19}b_{19} = 4.92$	$V_2b_2 = 4.30$	$Z_2 = 21.40$	$(V_{19}b_{19} - V_2b_2)Z_2 = 13.28$
a_3	$y_3 = 7.65$	58.52	$V_{18}b_{18} = 6.38$	$V_3b_3 = 5.69$	$Z_3 = 17.21$	$(V_{18}b_{18} - V_3b_3)Z_3 = 11.89$
a_4	$y_4 = 8.25$	68.06	$V_{17}b_{17} = 7.12$	$V_4b_4 = 6.41$	$Z_4 = 14.45$	$(V_{17}b_{17} - V_4b_4)Z_4 = 10.27$
a_5	$y_5 = 8.70$	75.69	$V_{16}b_{16} = 7.59$	$V_5b_5 = 6.92$	$Z_5 = 12.09$	$(V_{16}b_{16} - V_5b_5)Z_5 = 7.49$
a_6	$y_6 = 9.05$	81.90	$V_{15}b_{15} = 7.90$	$V_6b_6 = 7.30$	$Z_6 = 9.86$	$(V_{15}b_{15} - V_6b_6)Z_6 = 5.91$
a_7	$y_7 = 9.30$	86.49	$V_{14}b_{14} = 8.13$	$V_7b_7 = 7.60$	$Z_7 = 7.72$	$(V_{14}b_{14} - V_7b_7)Z_7 = 4.09$
a_8	$y_8 = 9.50$	90.25	$V_{13}b_{13} = 8.25$	$V_8b_8 = 7.85$	$Z_8 = 5.51$	$(V_{13}b_{13} - V_8b_8)Z_8 = 2.20$
a_9	$y_9 = 9.65$	93.12	$V_{12}b_{12} = 8.32$	$V_9b_9 = 8.05$	$Z_9 = 3.37$	$(V_{12}b_{12} - V_9b_9)Z_9 = 0.91$
a_{10}	$y_{10} = 9.75$	95.06	$V_{11}b_{11} = 8.31$	$V_{10}b_{10} = 8.20$	$Z_{10} = 1.12$	$(V_{11}b_{11} - V_{10}b_{10})Z_{10} = 0.12$
	81.45 2 162.90 $\Sigma(y) = 162.90$	700.61 2 $1401.22 = \Sigma(y^2)$	66.87 $R = \Sigma(vb)$ $= 66.87$ 62.32 $R = 129.19$	62.32	Sum = 56.16 $r = \frac{Sum}{R} = \frac{56.16}{129.19} = .4345$ ft. that R acts to the left of center line AB.	

TABLE V. CALCULATIONS FOR LOCATION OF TRIAL T AND TRIAL T_1.

vd Ordinates of Type vd of Triangle $v_{20}nv_1$	vd	Triangle $v_{20}nv_1$		Triangle v_1nn_1	
		z Lever Arm About AB.	Moments About Center Line AB.	nd Ordinates of Type nd of Triangle v_1nn_1	
$v_{20}n = 6.46$	$v_1 d_1 = 0.00$	$z_1 = 30.10$	$(v_{20}n - v_1 d_1)z_1 = 193.90$	$v_1 n_1 = 6.46$	$n d_{20} = 0.00$
$v_{19} d_{19} = 5.51$	$v_2 d_2 = 0.92$	$z_2 = 21.40$	$(v_{19} d_{19} - v_2 d_2)z_2 = 98.30$	$n_2 d_2 = 5.54$	$n_{19} d_{19} = 0.95$
$v_{18} d_{18} = 5.07$	$v_3 d_3 = 1.39$	$z_3 = 17.21$	$(v_{18} d_{18} - v_3 d_3)z_3 = 63.40$	$n_3 d_3 = 5.07$	$n_{18} d_{18} = 1.39$
$v_{17} d_{17} = 4.73$	$v_4 d_4 = 1.69$	$z_4 = 14.45$	$(v_{17} d_{17} - v_4 d_4)z_4 = 43.40$	$n_4 d_4 = 4.77$	$n_{17} d_{17} = 1.73$
$v_{16} d_{16} = 4.52$	$v_5 d_5 = 1.93$	$z_5 = 12.09$	$(v_{16} d_{16} - v_5 d_5)z_5 = 31.30$	$n_5 d_5 = 4.53$	$n_{16} d_{16} = 1.84$
$v_{15} d_{15} = 4.30$	$v_6 d_6 = 2.09$	$z_6 = 9.86$	$(v_{15} d_{15} - v_6 d_6)z_6 = 21.80$	$n_6 d_6 = 4.37$	$n_{15} d_{15} = 2.16$
$v_{14} d_{14} = 4.05$	$v_7 d_7 = 2.40$	$z_7 = 7.72$	$(v_{14} d_{14} - v_7 d_7)z_7 = 12.73$	$n_7 d_7 = 4.06$	$n_{14} d_{14} = 2.41$
$v_{13} d_{13} = 3.83$	$v_8 d_8 = 2.63$	$z_8 = 5.51$	$(v_{13} d_{13} - v_8 d_8)z_8 = 6.61$	$n_8 d_8 = 3.83$	$n_{13} d_{13} = 2.63$
$v_{12} d_{12} = 3.59$	$v_9 d_9 = 2.88$	$z_9 = 3.37$	$(v_{12} d_{12} - v_9 d_9)z_8 = 2.39$	$n_9 d_9 = 3.58$	$n_{12} d_{12} = 2.87$
$v_{11} d_{11} = 3.37$	$v_{10} d_{10} = 3.11$	$z_{10} = 1.12$	$(v_{11} d_{11} - v_{10} d_{10})z_{10} = 0.29$	$n_{10} d_{10} = 3.35$	$n_{11} d_{11} = 3.09$
$\Sigma(vd) = 64.47$ = Trial T		Sum = 474.12 $\frac{\text{Sum}}{\text{Trial } T} = \frac{474.12}{64.47} = 7.35$ Trial T acts 7.35' to left of ℄ AB $t = r + l$ $7.35 = .4345 + l$		$\Sigma(nd) = 64.68 = $ Trial T_1 Trial T_1 acts 7.35' to right of ℄ AB.	

TABLE VI DETERMINATION OF $\Sigma(mb.y)$ of EQUILIBRIUM POLYGON.

1	2	3	4	5
mb Ordinates of Type mb of Eq. Poly.	mb	Algebraic Sum of Col. 1 + Col. 2.	y Length of ordinates type y	(mb.y) (Col. 3 x Col. 4)
$m_1 b_1 = -6.07$	$m_{20} b_{20} = -6.85$	−12.92	3.15	− 40.69
$m_2 b_2 = -1.95$	$m_{19} b_{19} = -1.85$	− 3.80	6.45	− 24.25
$m_3 b_3 = -0.54$	$m_{18} b_{18} = -0.34$	− 0.88	7.65	− 6.73
$m_4 b_4 = +0.20$	$m_{17} b_{17} = +0.45$	+ 0.65	8.25	+ 5.35
$m_5 b_5 = +0.65$	$m_{16} b_{16} = +0.95$	+ 1.60	8.70	+ 13.91
$m_6 b_6 = +1.00$	$m_{15} b_{15} = +1.30$	+ 2.30	9.05	+ 20.81
$m_7 b_7 = +1.25$	$m_{14} b_{14} = +1.60$	+ 2.85	9.30	+ 26.75
$m_8 b_8 = +1.45$	$m_{13} b_{13} = +1.75$	+3.20	9.50	+30.20
$m_9 b_9 = +1.60$	$m_{12} b_{12} = +1.80$	+3.40	9.65	+32.80
$m_{10} b_{10} = +1.75$	$m_{11} b_{11} = +1.85$	+3.60	9.75	+35.06
Ordinates mb when above true closing line mm are regarded as + and when below as −.		+ 17.60 −17.60 0.00		+ 93.21 = Sum = $\Sigma(mb.y)$

TABLE VII. DETERMINATION OF ORDINATES OF ARCH PRESSURE LINE.

1	2	3	4	5
mb $\Sigma(mb)=0$	$\frac{\Sigma(y^2) - e_1\Sigma(y)}{\Sigma(mb.y)}$ $=.786$	kc Ordinates from Line kk to points c of Pressure Line of Arch (Col.1xCol.2) $\Sigma(kc)=0$	ka Ordinates from Line kk to points a on Neutral Axis $\Sigma(ka)=0$	ac Ordinates from points a to c of Arch $\Sigma(ac)=0$
$m_1 b_1 = -6.07$	0.786	−4.85	−5.09	+0.24
$m_2 b_2 = -1.95$	"	−1.53	−1.63	+0.10
$m_3 b_3 = -0.54$	"	−0.42	−0.45	+0.03
$m_4 b_4 = -0.20$	"	+0.16	+0.12	+0.04
$m_5 b_5 = +0.65$	"	+0.51	+0.57	−0.06
$m_6 b_6 = +1.00$	"	+0.78	+0.90	−0.12
$m_7 b_7 = +1.25$	"	+0.98	+1.17	−0.19
$m_8 b_8 = +1.45$	"	+1.18	+1.38	−0.20
$m_9 b_9 = +1.60$	"	+1.29	+1.51	−0.22
$m_{10} b_{10} = +1.75$	"	+1.38	+1.60	−0.22
$m_{11} b_{11} = +1.85$	"	+1.45	+1.55	−0.10
$m_{12} b_{12} = +1.80$	"	+1.42	+1.45	−0.03
$m_{13} b_{13} = +1.75$	"	+1.38	+1.38	0.00
$m_{14} b_{14} = +1.60$	"	+1.25	+1.15	+0.10
$m_{15} b_{15} = +1.30$	"	+1.02	+0.92	+0.10
$m_{16} b_{16} = +0.95$	"	+0.75	+0.57	+0.18
$m_{17} b_{17} = +0.45$	"	+0.35	+0.17	+0.18
$m_{18} b_{18} = -0.34$	"	−0.27	−0.50	+0.23
$m_{19} b_{19} = -1.85$	"	−1.45	−1.65	+0.20
$m_{20} b_{20} = -6.85$	"	−5.38	−5.12	−0.26
+17.60 −17.60 0.00		+13.90 −13.90 0.00	+14.44 −14.44 0.00	+1.40 −1.40 0.00

TABLE VIII. CALCULATION OF MOMENTS THRUSTS AND SHEARS.

Points	ac Ordinates from points a to c of arch	M Bending Moments in foot pounds $M = H(ac)$ $H = 44,529$	Thrusts = T Tang. Comps. at points a scaled from diag.	Shears Normal Comps. at points a scaled from diag.
1	+ 0.24	+ 11,400	50,200	2,050
2	+ 0.10	+ 4,450	46,400	1,250
3	+ 0.03	+ 1,337	45,500	1,750
4	+ 0.04	+ 1,783	45,200	1,750
5	− 0.06	− 2,675	45,000	1,450
6	− 0.12	− 5,350	44,900	1,300
7	− 0.19	− 8,470	44,800	1,050
8	− 0.20	− 8,900	44,700	400
9	− 0.22	− 9,810	44,700	150
10	− 0.22	− 9,810	44,600	600
11	− 0.10	− 4,450	44,500	1,250
12	− 0.03	− 1,337	44,500	1,350
13	0.00	0,000	44,500	1,400
14	+ 0.10	+ 4,450	44,600	1,200
15	+ 0.10	+ 4,450	44,700	1,400
16	+ 0.18	+ 8,020	45,000	1,200
17	+ 0.18	+ 8,020	45,300	650
18	+ 0.23	+ 10,200	45,700	500
19	+ 0.20	+ 8,900	46,900	700
20	− 0.26	− 11,600	51,500	5,450

TABLE IX. CALCULATION OF TANGENTIAL THRUST, T_t, DUE TO TEMPERATURE.

$\lambda : \frac{1}{2}N = \Phi = \frac{1}{2}\alpha$

$\lambda : 37.8 = \phi = 30.175$

$\phi = .798\,\lambda$

N = length of neutral axis = 75.6'

α = subtended central angle = 60.35°

Point	λ Distance from ℄ AB of span to points a on N. Ax.	ϕ $= .798\,\lambda$	cosine ϕ	T_t $= H_t \cos\phi$ $= 3505 \cos\phi$
1	31.10	24° – 49'	0.90766	3175
2	21.73	17° – 21'	0.95450	3340
3	17.38	13° – 52'	0.97086	3400
4	14.48	11° – 34'	0.97969	3430
5	12.13	9° – 42'	0.98570	3455
6	9.91	7° – 16'	0.99197	3470
7	7.71	6° – 9'	0.99424	3480
8	5.51	4° – 24'	0.99705	3490
9	4.31	3° – 26'	0.99821	3495
10	1.11	0° – 53'	0.99988	3500
11	1.11	0° – 53'	0.99988	3500
12	4.31	3° – 26'	0.99821	3495
13	5.51	4° – 24'	0.99705	3490
14	7.71	6° – 9'	0.99424	3480
15	9.91	7° – 16'	0.99197	3470
16	12.13	9° – 42'	0.99570	3455
17	14.48	11° – 34'	0.97969	3430
18	17.38	13° – 52'	0.97086	3400
19	21.73	17° – 21'	0.95450	3340
20	31.10	24° – 49'	0.90766	3175

TABLE X. QUANTITIES NECESSARY FOR FIBER STRESS CALCULATIONS.

Pt.	D rad. dp in ft	D^3	$\frac{1}{12}D^3$	$A_c = D \times 1$	$X_1 = \frac{D}{2}$	$D' = D-2G = D-.5$	$X_2 = \frac{D'}{2}$	X_2^2	nA_s const $= 20 \times .016$	M	T	H_t const $= 3505$	K_a	$M_t = H_t(K_a)$	$T_t = H_t \cos\phi$
1	2.43	14.34	1.19	2.43	1.21	1.93	.96	.92	.32	+ 11,400	50,200	3505	− 5.09	− 17,840	3175
2	1.79	5.73	0.48	1.79	.89	1.29	.64	.40	"	+ 4,450	46,400	"	− 1.63	− 5,740	3340
3	1.53	3.58	0.29	1.53	.76	1.03	.51	.26	"	+ 1,337	45,500	"	− 0.45	− 1,570	3400
4	1.37	2.57	0.21	1.37	.68	.87	.43	.18	"	+ 1,783	45,200	"	+ 0.12	+ 420	3430
5	1.34	2.40	0.20	1.34	.67	.84	.42	.17	"	− 2,675	45,000	"	+ 0.57	+ 2,000	3455
6	1.33	2.35	0.19	1.33	.66	.83	.41	.16	"	− 5,350	44,900	"	+ 0.90	+ 3,150	3470
7	"	"	"	"	"	"	"	"	"	− 8,470	44,800	"	+ 1.17	+ 4,110	3480
8	"	"	"	"	"	"	"	"	"	− 8,900	44,700	"	+ 1.38	+ 4,840	3490
9	"	"	"	"	"	"	"	"	"	− 9,810	44,700	"	+ 1.51	+ 5,300	3495
10	"	"	"	"	"	"	"	"	"	− 9,810	44,600	"	+ 1.60	+ 5,620	3500
11	"	"	"	"	"	"	"	"	"	− 4,450	44,500	"	+ 1.55	+ 5,440	3500
12	"	"	"	"	"	"	"	"	"	− 1,337	44,500	"	+ 1.45	+ 5,090	3495
13	"	"	"	"	"	"	"	"	"	0,000	44,500	"	+ 1.38	+ 4,840	3490
14	"	"	"	"	"	"	"	"	"	+ 4,450	44,600	"	+ 1.15	+ 4,030	3480
15	"	"	"	"	"	"	"	"	"	+ 4,450	44,700	"	+ 0.92	+ 3,220	3470
16	1.34	2.40	0.20	1.34	.67	.84	.42	.17	"	+ 8,020	45,000	"	+ 0.57	+ 2,000	3455
17	1.37	2.57	0.21	1.37	.68	.87	.43	.18	"	+ 8,020	45,300	"	+ 0.17	+ 597	3430
18	1.53	3.58	0.29	1.53	.76	1.03	.51	.26	"	+ 10,200	45,700	"	− 0.50	− 1,750	3400
19	1.79	5.73	0.48	1.79	.89	1.29	.64	.40	"	+ 8,900	46,900	"	− 1.65	− 5,790	3340
20	2.43	14.34	1.19	2.43	1.21	1.93	.96	.92	"	− 11,600	51,500	"	− 5.12	− 17,940	3175

TABLE XI. QUANTITIES NECESSARY FOR FIBER STRESS CALCULATIONS.

CALCULATIONS OF VALUES α AND β.

$\alpha = A_c + nA_s.$

$\beta = \frac{1}{12} D^3 + nA_s k_2^2.$

Point	1	2	3	4	5	6	7
	A_c	nA_s = constant	α = Col. 1 + Col. 2	$\frac{1}{12} D^3$	X_2^2	$nA_s \cdot X_2^2$ Col. 2 × Col. 5	β = Col. 4 + Col. 6
1	2.43	.32	2.75	1.19	.92	0.29	1.48
2	1.79	"	2.11	0.48	.40	.0.13	0.61
3	1.53	"	1.85	0.29	.26	.0.83	0.37
4	1.37	"	1.69	0.21	.18	.058	0.27
5	1.34	"	1.66	0.20	.17	.054	0.25
6	1.33	"	1.65	0.19	.16	.051	0.24
7	"	"	"	"	"	"	"
8	"	"	"	"	"	"	"
9	"	"	"	"	"	"	"
10	"	"	"	"	"	"	"
11	"	"	"	"	"	"	"
12	"	"	"	"	"	"	"
13	"	"	"	"	"	"	"
14	"	"	"	"	"	"	"
15	"	"	"	"	"	"	"
16	1.34	"	1.66	0.20	.17	.054	0.25
17	1.37	"	1.69	0.21	.18	.058	0.27
18	1.53	"	1.85	0.29	.26	.083	0.37
19	1.79	"	2.11	0.48	.40	.130	0.61
20	2.43	"	2.75	1.19	.92	.290	1.48

TABLE XII. STRESSES IN CONCRETE.

$$S_c = \frac{T}{A_c + nA_s} \pm \frac{Mx_1}{\frac{1}{12}D^3 + nA_sx_2^2} = \frac{T}{\alpha} \pm \frac{Mx_1}{\beta}$$

	1	2	3	4	5	6	7	8	9	10	11	12
Pts.	T	M	x_1	α	β	Mx_1	$\frac{T}{\alpha}$	$\frac{Mx_1}{\beta}$	Col. 7 + Col. 8 #/sq. ft. Upper Fiber	Col. 9 ÷ 144 #/sq. in. Upper Fiber	Col. 7 − Col. 8 #/sq. ft. Lower Fiber	Col. 11 ÷ 144 #/sq. in. Lower Fiber
1	50,200	+ 11,400	1.21	2.75	1.48	+ 13,800	+ 18,240	+ 9,330	+ 27,570	+ 191	+ 8,910	+ 62
2	46,400	+ 4,450	.89	2.11	.61	+ 3,960	+ 21,950	+ 6,490	+ 28,440	+ 197	+ 15,460	+ 107
3	45,500	+ 1,357	.76	1.85	.37	+ 1,010	+ 24,550	+ 2,715	+ 27,265	+ 189	+ 21,835	+ 151
4	45,200	+ 1,783	.68	1.69	.27	+ 1,210	+ 26,700	+ 4,480	+ 31,180	+ 216	+ 22,220	+ 154
5	45,000	− 2,675	.67	1.66	.25	− 1,790	+ 27,100	− 7,160	+ 19,940	+ 138	+ 34,760	+ 241
6	44,900	− 5,350	.66	1.65	.24	− 3,530	+ 27,250	− 14,700	+ 12,550	+ 87	+ 41,950	+ 291
7	44,800	− 8,470	"	"	"	− 5,590	+ 27,150	− 23,300	+ 3,850	+ 26	+ 50,450	+ 350
8	44,700	− 8,900	"	"	"	− 5,870	+ 27,100	− 24,400	+ 2,700	+ 19	+ 51,500	+ 358
9	44,700	− 9,810	"	"	"	− 6,480	+ 27,100	− 26,900	+ 200	+ 2	+ 54,000	+ 375
10	44,600	− 9,810	"	"	"	− 6,480	+ 27,050	− 26,900	+ 150	+ 1	+ 53,950	+ 374
11	44,500	− 4,450	"	"	"	− 2,940	+ 26,950	− 12,250	+ 14,700	+ 102	+ 39,200	+ 272
12	44,500	− 1,337	"	"	"	− 882	+ 26,950	− 3,675	+ 23,275	+ 162	+ 30,625	+ 212
13	44,500	− 0,000	"	"	"	− 000	+ 26,950	− 0,000	+ 26,950	+ 186	+ 26,950	+ 187
14	44,600	+ 4,450	"	"	"	+ 2,940	+ 27,050	+ 12,250	+ 39,300	+ 272	+ 14,800	+ 103
15	44,700	+ 4,450	"	"	"	+ 2,940	+ 27,100	+ 12,250	+ 39,350	+ 272	+ 14,860	+ 103
16	45,000	+ 8,020	.67	1.66	.25	+ 5,500	+ 27,100	+ 21,970	+ 49,070	+ 340	+ 5,130	+ 35
17	45,300	+ 8,020	.68	1.69	.27	+ 5,450	+ 26,800	+ 20,200	+ 47,000	+ 326	+ 6,600	+ 46
18	45,700	+ 10,200	.76	1.85	.37	+ 7,750	+ 24,700	+ 20,950	+ 45,650	+ 317	+ 3,750	+ 25
19	46,900	+ 8,900	.89	2.11	.61	+ 7,920	+ 22,200	+ 12,950	+ 35,150	+ 244	+ 9,250	+ 64
20	51,500	− 11,600	1.21	2.75	1.48	− 14,050	+ 18,730	− 9,490	+ 9,240	+ 64	+ 28,220	+ 195

+ Compression. − Tension.

TABLE XIII. STRESSES IN STEEL

$$S_s = \left\{ \frac{T}{A_c + nA_s} \pm \frac{Mx_2}{\frac{1}{12}D^3 + nA_s x_2^2} \right\} 20 = \left\{ \frac{T}{\alpha} \pm \frac{Mx_2}{\beta} \right\} 20$$

	1	2	3	4	5	6	7	8	9	10	11	12
Pts.	T	M	x_2	α	β	Mx_2	$\frac{T}{\alpha}$	$\frac{Mx_2}{\beta}$	(Col.7+Col.8)20 # /sq. ft. Up. Fiber	Col. 9 ÷ 144 # /sq. in. Up. Fiber	(Col.7-Col.8)20 # /sq. ft. Lower Fiber	Col. 11 ÷ 144 # /sq. in. Lower Fiber
1	50,200	+ 11,400	.96	2.75	1.48	+10,950	+ 18,240	+ 7,380	+ 512,400	+ 3,350	+ 217,200	+ 1,505
2	46,400	+ 4,450	.64	2.11	0.61	+ 2,850	+ 21,950	+ 4,650	+ 532,000	+ 3,690	+ 346,000	+ 2,405
3	45,500	+ 1,337	.51	1.85	.37	+ 680	+ 24,550	+ 1,835	+ 527,700	+ 3,650	+ 454,300	+ 3,150
4	45,200	+ 1,783	.43	1.69	.27	+ 765	+ 26,700	+ 1,830	+ 570,600	+ 3,950	+ 497,400	+ 3,450
5	45,000	- 2,675	.42	1.66	.25	- 1,120	+ 27,100	- 4,480	+ 452,400	+ 3,130	+ 631,600	+ 4,380
6	44,900	- 5,350	.41	1.65	.24	- 2,195	+ 27,250	- 9,130	+ 362,400	+ 2,510	+ 727,600	+ 5,040
7	44,800	- 8,470	"	"	"	- 3,470	+ 27,150	- 14,450	+ 254,000	+ 1,760	+ 832,000	+ 5,780
8	44,700	- 8,900	"	"	"	- 3,550	+ 27,100	- 14,800	+ 246,000	+ 1,710	+ 838,000	+ 5,820
9	44,700	- 9,810	"	"	"	- 4,030	+ 27,100	- 16,800	+ 406,000	+ 2,820	+ 878,000	+ 6,090
10	44,600	- 9,810	"	"	"	- 4,030	+ 27,050	- 16,800	+ 405,000	+ 2,810	+ 877,000	+ 6,080
11	44,500	- 4,450	"	"	"	- 1,825	+ 26,950	- 7,580	+ 387,400	+ 2,690	+ 690,600	+ 4,780
12	44,500	- 1,337	"	"	"	- 545	+ 26,950	- 2,270	+ 493,600	+ 3,430	+ 584,400	+ 4,060
13	44,500	0,000	"	"	"	000	+ 26,950	0,000	+ 539,000	+ 3,740	+ 599,000	+ 4,160
14	44,600	+ 4,450	"	"	"	+ 1,825	+ 27,050	+ 7,580	+ 692,600	+ 4,800	+ 389,400	+ 2,700
15	44,700	+ 4,450	"	"	"	+ 1,825	+ 27,100	+ 7,580	+ 693,600	+ 4,810	+ 390,400	+ 2,710
16	45,000	+ 8,020	.42	1.66	.25	+ 3,370	+ 27,100	+ 13,450	+ 811,000	+ 5,630	+ 273,000	+ 1,895
17	45,300	+ 8,020	.43	1.69	.27	+ 3,450	+ 26,800	+ 12,750	+ 791,000	+ 5,493	+ 281,000	+ 1,950
18	45,700	+ 10,200	.51	1.85	.37	+ 5,200	+ 24,700	+ 14,050	+ 775,000	+ 5,370	+ 212,000	+ 1,470
19	46,900	+ 8,900	.64	2.11	.61	+ 5,690	+ 22,200	+ 9,320	+ 630,400	+ 4,370	+ 257,600	+ 1,785
20	51,500	- 11,600	.96	2.75	1.48	- 11,120	+ 18,730	- 7,520	+ 224,200	+ 1,550	+ 525,000	+ 3,650

TABLE XIV. STRESSES DUE TO TEMPERATURE. CONCRETE.

$$S_{ct} = \frac{T_t}{A_c + nA_s} \pm \frac{M_t x_1}{\frac{1}{12}D^3 + nA_s x_2^2} = \frac{T_t}{\alpha} \pm \frac{M_t x_1}{\beta}$$

	1	2	3	4	5	6	7	8	9	10	11	12
Pts.	T_t	M_t	x_1	α'	β	$M_t x_1$	$\frac{T_t}{\alpha}$	$\frac{M_t x_1}{\beta}$	Col. 7 + Col. 8 #/sq. ft.	Col. 9 ÷ 144 #/sq. in.	Col. 7 − Col. 8 #/sq. ft.	Col. 11 ÷ 144 #/sq. in.
1	3175	−17,840	1.21	2.75	1.48	−21,600	+1,152	−14,600	−13,448	−93	+15,752	+109
2	3340	−5,740	.89	2.11	.61	−5,120	+1,560	−8,380	−6,820	−47	+9,940	+69
3	3400	−1,570	.76	1.85	.37	−1,190	+1,840	−3,220	−1,380	−9	+5,060	+39
4	3430	+420	.68	1.69	.27	+284	+2,015	+1,050	+2,065	+14	+965	+7
5	3455	+2,000	.67	1.66	.25	+1,340	+2,070	+5,360	+7,430	+51	−3,290	−21
6	3470	+3,150	.66	1.65	.24	+2,080	+2,100	+8,670	+10,770	+74	−6,570	−45
7	3480	+4,110	"	"	"	+2,710	+2,110	+11,600	+13,710	+95	−9,490	−66
8	3490	+4,840	"	"	"	+3,190	+2,114	+13,600	+15,714	+109	−11,486	−79
9	3495	+5,300	"	"	"	+3,500	+2,117	+14,600	+16,717	+116	−12,483	−87
10	3500	+5,620	"	"	"	+3,700	+2,120	+15,400	+17,520	+121	−13,280	−92
11	3500	+5,440	"	"	"	+3,490	+2,120	+14,500	+16,620	+115	−12,380	−85
12	3495	+5,090	"	"	"	+3,360	+2,117	+14,000	+16,117	+112	−11,883	−82
13	3490	+4,840	"	"	"	+3,190	+2,114	+13,600	+15,714	+109	−11,486	−79
14	3480	+4,030	"	"	"	+2,660	+2,110	+11,100	+13,220	+92	−8,990	−62
15	3470	+3,220	"	"	"	+2,120	+2,100	+8,830	+10,930	+76	−6,730	−47
16	3455	+2,000	.67	1.66	.25	+1,340	+2,070	+5,360	+7,430	+51	−3,290	−22
17	3430	+597	.68	1.69	.27	+406	+2,015	+1,510	+3,525	+24	+505	+4
18	3400	−1,750	.76	1.85	.37	−1,360	+1,840	−3,680	−1,840	−13	+5,520	+38
19	3340	−5,790	.89	2.11	.61	−5,140	+1,560	−8,430	−6,870	−47	+9,990	+69
20	3175	−17,940	1.21	2.75	1.48	−21,700	+1,562	−14,600	−13,038	−90	+16,162	+112

TABLE XII. STRESSES DUE TO TEMPERATURE. STEEL.

$$S_{st} = \left\{ \frac{T_t}{A_c + nA_s} \pm \frac{M_t x_2}{\frac{1}{12}D^3 + nA_s x_2^2} \right\} 20 = \left\{ \frac{T_t}{\alpha} \pm \frac{M_t x_2}{\beta} \right\} 20$$

Pts.	1	2	3	4	5	6	7	8	9	10	11	12
	T_t	M_t	x_2	α	β	$M_t x_2$	$\frac{T_t}{\alpha}$	$\frac{M_t x_2}{\beta}$	(Col. 7 + Col. 8)20 #/sq. ft.	Col. 9 ÷ 144 #/sq. in.	(Col. 7 - Col. 8)20 #/sq. ft	Col. 11 ÷ 144 #/sq. in.
1	+ 3175	- 17,840	.96	2.75	1.48	- 17,100	+ 1,152	- 11,500	- 206,960	- 1430	+ 253,040	+ 1760
2	+ 3340	- 5,740	.64	2.11	.61	- 3,670	+ 1,560	- 6,040	- 89,600	- 620	+ 152,000	+ 1050
3	+ 3400	- 1,570	.51	1.85	.37	- 801	+ 1,840	- 2,170	- 6,600	- 47	+ 60,200	+ 418
4	+ 3430	+ 420	.43	1.69	.27	+ 180	+ 2,015	+ 667	+ 53,640	+ 372	+ 26,960	+ 187
5	+ 3455	+ 2,000	.42	1.66	.25	+ 840	+ 2,070	+ 3,360	+ 108,600	+ 752	- 25,800	- 179
6	+ 3470	+ 3,150	.41	1.65	.24	+ 1,290	+ 2,100	+ 5,380	+ 149,600	+ 1030	- 65,600	- 456
7	+ 3480	+ 4,110	"	"	"	+ 1,680	+ 2,110	+ 7,000	+ 182,200	+ 1260	- 97,800	- 678
8	+ 3490	+ 4,840	"	"	"	+ 1,980	+ 2,114	+ 8,250	+ 207,280	+ 1440	- 122,720	- 841
9	+ 3495	+ 5,300	"	"	"	+ 2,170	+ 2,117	+ 9,040	+ 223,140	+ 1550	- 138,460	- 960
10	+ 3500	+ 5,620	"	"	"	+ 2,310	+ 2,120	+ 9,630	+ 235,000	+ 1630	- 150,200	- 1040
11	+ 3500	+ 5,440	"	"	"	+ 2,230	+ 2,120	+ 9,290	+ 228,200	+ 1510	- 143,400	- 994
12	+ 3495	+ 5,090	"	"	"	+ 2,090	+ 2,117	+ 8,720	+ 216,740	+ 1510	- 132,060	- 917
13	+ 3490	+ 4,840	"	"	"	+ 1,980	+ 2,114	+ 8,260	+ 207,480	+ 1440	- 122,920	- 841
14	+ 3480	+ 4,030	"	"	"	+ 1,650	+ 2,110	+ 6,980	+ 181,800	+ 1260	- 97,400	- 676
15	+ 3470	+ 3,220	"	"	"	+ 1,320	+ 2,100	+ 5,510	+ 152,200	+ 1050	- 68,200	- 473
16	+ 3455	+ 2,000	.42	1.66	.25	+ 840	+ 2,070	+ 3,360	+ 108,600	+ 754	- 25,800	- 179
17	+ 3430	+ 597	.43	1.69	.27	+ 285	+ 2,015	+ 1,060	+ 61,500	+ 427	+ 19,100	+ 133
18	+ 3400	- 1,750	.51	1.85	.37	- 892	+ 1,840	- 2,410	- 10,400	- 72	+ 85,000	+ 590
19	+ 3340	- 5,790	.64	2.11	.61	- 3,710	+ 1,560	- 6,080	- 90,400	- 627	+ 162,800	+ 1060
20	+ 3175	- 17,940	.96	2.75	1.48	- 17,200	+ 1,152	- 11,600	- 208,960	- 1450	+ 255,040	+ 1770

TABLE XVI. LOCATION OF FIBER STRESS (UPPER OR LOWER FIBER) DUE TO TEMPERATURE. -

	When ka is +		When ka is −	
	Upper Fiber	Lower Fiber	Upper Fiber	Lower Fiber
Rise in Temperature	Tension	Compression	Compression	Tension
Fall in Temperature	Compression	Tension	Tension	Compression

Pts.	ka algebraic sign	Concrete				Steel			
		+40°F. (Rise)		−40°F. (Fall)		+40°F.		−40°F.	
		Up. Fiber	Lower Fiber	U. F.	L. F.	U. F.	L. F.	U.F.	L. F.
1	−	+109	−93	−93	+109	+1760	−1430	−1430	+1760
2	−	+69	−47	−47	+69	+1050	−620	−620	+1050
3	−	+39	−9	−9	+39	+418	−47	−47	+418
4	+	+7	+14	+14	+7	+187	+372	+372	+187
5	+	−21	+51	+51	−21	−179	+752	+752	−179
6	+	−45	+74	+74	−45	−456	+1030	+1030	−456
7	+	−66	+95	+95	−66	−678	+1260	+1260	−678
8	+	−79	+109	+109	−79	−841	+1440	+1440	−841
9	+	−87	+116	+116	−87	−960	+1550	+1550	−960
10	+	−98	+121	+121	−92	−1040	+1630	+1630	−1040
11	+	−85	+115	+115	−85	−994	+1510	+1510	−994
12	+	−82	+112	+112	−82	−917	+1510	+1510	−917
13	+	−79	+109	+109	−79	−841	+1440	+1440	−841
14	+	−62	+92	+92	−62	−676	+1260	+1260	−676
15	+	−49	+76	+76	−49	−473	+1050	+1050	−473
16	+	−22	+51	+51	−22	−179	+754	+754	−179
17	+	+4	+24	+24	+4	+133	+427	+427	+133
18	−	+38	−13	−13	+38	+590	−72	−72	+590
19	−	+69	−47	−47	+69	+1060	−627	−627	+1060
20	−	+112	−90	−90	+112	+1770	−1450	−1450	+1770

TABLE XVIII. STRESSES COMBINED TO OBTAIN MAXIMUM FIBER STRESS.

	Concrete		Steel		Stresses due to change in temperature								Maximum fiber stress (Temp. stresses included) (#/sq.in.)			
					Concrete				Steel							
					+40°F.		−40°F.		+40°F.		−40°F.		Concrete		Steel	
Pts	U.F. Extrados	L.F. Intrados	U.F. Ex.	L.F. In.	U.F.	L.F.	U.F.	L.F.	U.F.	L.F.	U.F.	L.F.	U.F.	L.F.	U.F.	L.F.
1	+191	+62	+3550	+1505	+109	−93	−93	+109	+1760	−1430	−1430	+1760	+300	−31	+5310	+75
2	+197	+107	+3690	+2405	+69	−47	−47	+69	+1050	−620	−620	+1050	+266		+4740	
3	+189	+151	+3650	+3150	+39	−9	−9	+39	+418	−47	−47	+418	+228		+4068	
4	+216	+154	+3950	+3450	+7	+14	+14	+7	+187	+372	+372	+187	+230		+4137	
5	+138	+241	+3130	+4380	−21	+51	+51	−21	−179	+752	+752	−179		+292		+5132
6	+87	+291	+2510	+5040	−45	+74	+74	−45	−456	+1030	+1030	−456		+365		+6070
7	+26	+350	+1760	+5780	−66	+95	+95	−66	−678	+1260	+1260	−678		+445		+7040
8	+19	+358	+1710	+5820	−79	+109	+109	−79	−841	+1440	+1440	−841		+467		+7260
9	+2	+375	+2820	+6090	−87	+116	+116	−87	−960	+1550	+1550	−960		+491		+7640
10	+1	+374	+2810	+6080	−92	+121	+121	−92	−1040	+1630	+1630	−1040		+495		+6710
11	+102	+272	+2690	+4780	−85	+115	+115	−85	−994	+1510	+1510	−994		+387		+6290
12	+162	+212	+3430	+4060	−82	+112	+112	−82	−917	+1510	+1510	−917		+324		+5570
13	+186	+187	+3740	+4160	−79	+109	+109	−79	−841	+1440	+1440	−841		+276		+5600
14	+272	+103	+4800	+2700	−62	+92	+92	−62	−676	+1260	+1260	−676	+364		+6060	
15	+272	+103	+4810	+2710	−49	+76	+76	−49	−473	+1050	+1050	−473	+348		+5860	
16	+340	+35	+5630	+1895	−22	+51	+51	−22	−179	+754	+754	−179	+391		+6384	
17	+326	+46	+5493	+1950	+4	+24	+24	+4	+133	+427	+427	+133	+350		+5920	
18	+317	+25	+5370	+1470	+38	−13	−13	+38	+590	−72	−72	+590	+355		+5960	
19	+244	+64	+4370	+1785	+69	−47	−47	+69	+1060	−627	−627	+1060	+313		+5430	
20	+64	+195	+1550	+3650	+112	−90	−90	+112	+1770	−1450	−1450	+1770	−26	+307	+100	+5420